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CHAPTER 1
Introduction

INTRODUCTION

The word 'Hydraulics' has been derived from a Greek word 'Hudour' which means water. Hydraulics is that branch of engineering which deals with water at rest or in motion. It deals mainly with the practical problems of flow of water and is based upon the results obtained from experiments. It provides various principles to solve practical problems in water supply, irrigation engineering, water power and hydraulic machines.

Pneumatics is that branch of engineering which deals with the action of compressed air or any other gas in operating various machines and equipment's.

FLUID Fluid may be defined as a substance which is capable of flowing and offers practically no resistance to the change of shape**.**

A fluid has no definite shape of its own, but takes the shape of the containing vessel. A fluid has no tensile strength or very little of it and it can resist compressive forces when it is kept in a container, When subjected to shearing force, a fluid deforms continuously as long—as force is applied. For mechanical analysis, a fluid is considered to be continuum i.e. a continuous distribution of matter with no void or empty space. Some of the examples of fluids are water, oil, air, gases and vapours.
Fluids may be classified as follow:
1. Liquids,
2. Gases including vapours.
1. **Liquids**: Liquids occupy a definite volume and are not affected appreciably by change in temperature or compression. Water, oil, honey, glycerine, paint, blood etc. are the examples of liquids.
2**. Gases including vapours**: Gases and vapours do not occupy a definite volume, but take the shape and volume of vessels containing them. Gases and vapours readily respond to change • temperature. These are capable of being compressed to a considerably small volume under high pressure.

TYPES OF FLUIDS
The fluids may be classified into the following two categories
1. Ideal fluids,
2. Real fluids.

1. Ideal Fluids: the fluids which are incompressible and have no viscosity and surface tension. These are only imaginary fluids and do not exist.
However, air and water may be considered as ideal fluids without much error.

2. Real Fluids: The fluids which possess properties such as viscosity, surface tension and compressibility are called real fluids. The fluids actually available in nature are real fluids. These fluids offer a certain amount of resistance when these are set in motion.

These are further subdivided into the following categories

(i) Newtonian fluids,
(ii) Non-Newtonian fluids,
(iii) Ideal plastic fluids,
(iv) Thixotropic fluids.

(i) **Newtonian Fluids**: The fluids in which shear stress is directly proportional to the rate of shear strain (or velocity gradient) are called Newtonian fluids. These fluids follow Newton's law of viscosity.
(ii) **Non-Newtonian Fluids**: The fluids in which shear stress is not proportional to the rate of shear strain (or velocity gradient) are called non-Newtonian fluids.
(iii) **Ideal Plastic Fluids**: The fluids in which shear stress is more than yield stress value r and shear stress is directly proportional to the rate of shear strain (velocity gradient) are called ideal plastic fluids.
(iv) **Thixotropic Fluids**: The fluids in which shear stress is more than Yield stress value and shear stress is not proportional to the rate of shear strain (or velocity gradient) called thixotropic fluids. e.g. printer's ink.

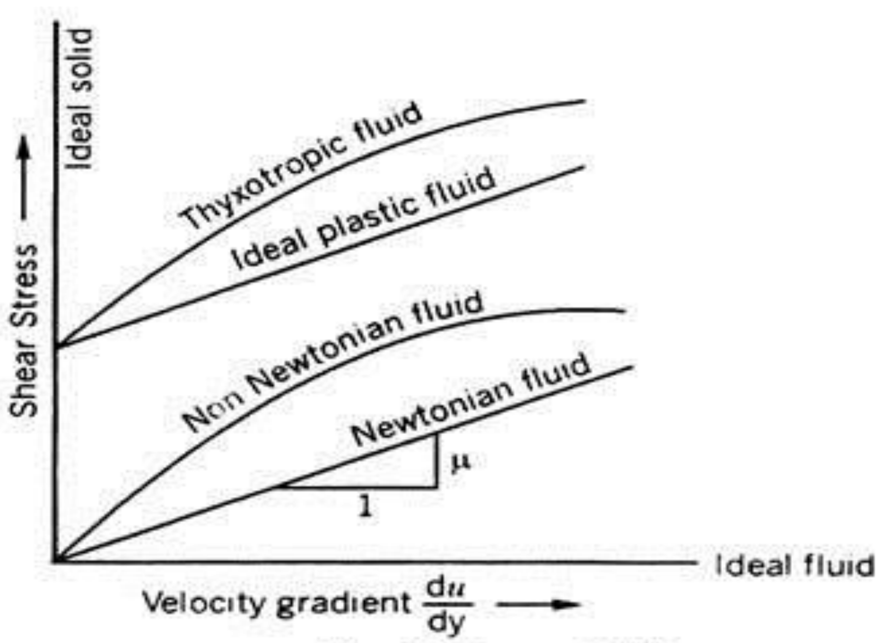


Fig. 1.5. Types of fluids

PROPERTIES OF FLUIDS

Some of the important properties of fluids are as follow

(i) Mass density,
(ii) Specific weight
(iii) Specific volume
(iv) Specific gravity
(v) Viscosity
(vi) Vapour pressure
(vii) Cohesion
(viii) Adhesion
(ix) Surface tension
(x) Capillarity

(xi) Compressibility

Mass Density

Mass density of fluid may be defined as mass of fluid per unit volume. It is generally by p (Rho). Its S.I. unit is kg/m3.

$$\text{Density}, \rho = \frac{\text{Mass}}{\text{Volume}} \; \frac{\text{kg}}{\text{m}^3}$$

The mass density of water is taken as 1000 kg/m3 at 4°C.

Specific Weight

Specific weight of fluid may be defined as weight of fluid per unit volume It is denoted by w. Its SI unit is N/m3. Specific weight varies from place to place due to the change of acceleration due to gravity (g).

Mathematically,

$$\text{Specific weight}, w = \frac{\text{Weight}}{\text{Volume}} \; \frac{\text{N}}{\text{m}^3}$$

Specific weight depends upon mass density and gravitational acceleration. Since gravitational acceleration varies from place to place, therefore, specific weight also varies from place to place. Specific weight also decreases with the increase in temperature. It increases with increase in pressure. However, the specific weight of water is taken as 9810 N/m3 at 4°C.

Specific Volume

Specific volume may be defined as the volume occupied by fluid per unit mass.it is generally denoted by v. its SI unit is m ^3/kg.

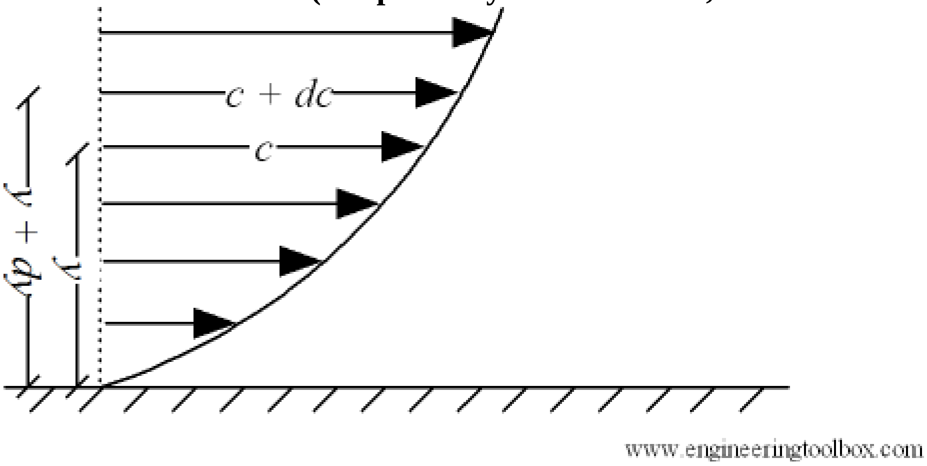
Specific volume is reciprocal of mass density.

Specific gravity,

Specific gravity is the ratio of the density (mass of a unit volume) of a substance to the density of a given reference material. Specific gravity for liquids is nearly always measured with respect to water at its densest (at 4 °C or 39.2 °F); for gases, air at room temperature (20 °C or 68 °F) is the reference. The term "relative density" is often preferred in scientific usage. It is defined as a ratio of density of particular substance with that of water.

Viscosity

Viscosity is a measure of a fluid's resistance to flow. It describes the internal friction of a moving fluid. A fluid with large viscosity resists motion because its molecular makeup gives it a lot of internal friction.



A fluid with low viscosity flows easily because its molecular makeup results in very little friction when it is in motion.
Gases also have viscosity, although it is a little harder to notice it in ordinary circumstances.

Kinematic viscosity
The ratio of viscosity to mass density of Fluid is called kinematic viscosity.
SI unit m^2/s. Another unit of KV is Stroke.
1 m^2/s=10000 strokes

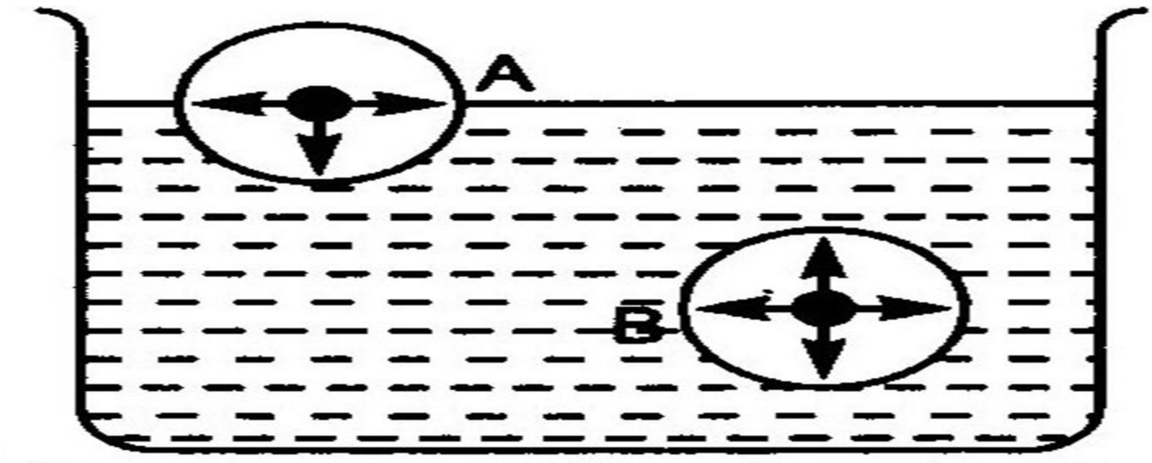
Compressibility
Compressibility of a fluid may be defined the property by virtue of which the fluid undergoes a change in volume under the action of external pressure. All the fluids can be compressed by the application of external pressure and when the pressure is removed, the compressed volumes of fluids expand to their on volumes, Thus fluids also possess elastic characteristics just like elastic solids.
The variation in the volume of water with the variation of pressure is so small that for practical purposes, it is neglected. Thus water is considered as incompressible fluid, Compressibility of fluid may be expressed as the reciprocal of bulk modulus of elasticity (K).
Bulk modulus of elasticity (K) may be defined as the ratio of compressive stress to volumetric strain.

Cohesion
Cohesion is the property of liquid by virtue of which it can withstand tension, property of liquid is due to the intermolecular attraction between the molecules of the liquid. The property of surface tension is also due to cohesion. The droplet of water hanging down the tap keeps its entity together due to the property of cohesion

Adhesion
Adhesion is the property of liquid by virtue of which it adheres (stick.) to the solid body with which it is in contact. Whereas cohesion is due to its inter-molecular attraction between the molecules of the liquid, adhesion is due to the forces of attraction between the molecules of the liquid and the molecules of the solid body, A droplet of water before falling from the tip of the finger exhibits the property of adhesion,

Surface Tension
The property of liquid by virtue of which the free surface of the liquid acts as a

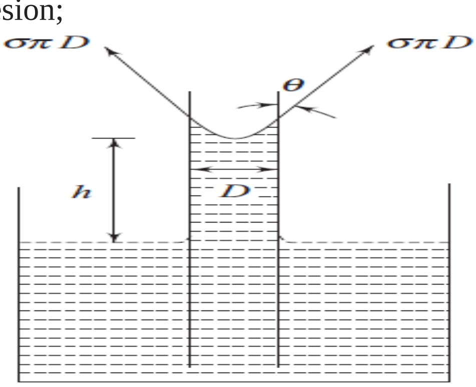
stretched elastic membrane capable of bearing a slight amount of tension is called surface tension.



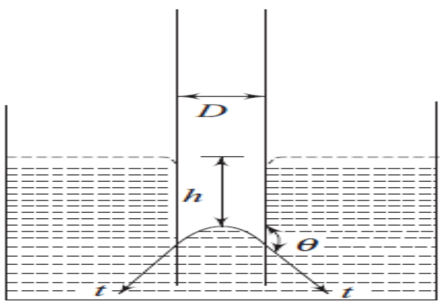
Mathematically, the surface tension may be defined as the force required unit length of Film in equilibrium. Its S.I unit is N/m
The property of surface tension can be described by slowly placing a steel needle on the surface of water in the horizontal position. The needle will continue floating on the surface of water exhibiting a little depression on the surface: The property of surface tension is due to the cohesion between the particles of the liquid.

Capillarity

Capillarity is the phenomenon by which a liquid rises up or falls down in a thin glass tube in comparison to the general liquid level in the vessel, when the glass tube is dipped into the mass of liquid. The rise of liquid is known as capillary rise whereas the fail of liquid Is known as capillary depression. It is generally expressed in terms of mm or cm of liquid. The phenomenon of capillarity is due to the effect of cohesion and adhesion of liquid particles, (If the cohesion between the liquid particles is less than the liquid in the tube will rise to the general more than adhesion, then the liquid in the tube will go down the liquid. the adhesion with the glass tube, then level of the liquid, If the cohesion;



Capillary rise
Adhesion > cohesion
liquid wets the surface



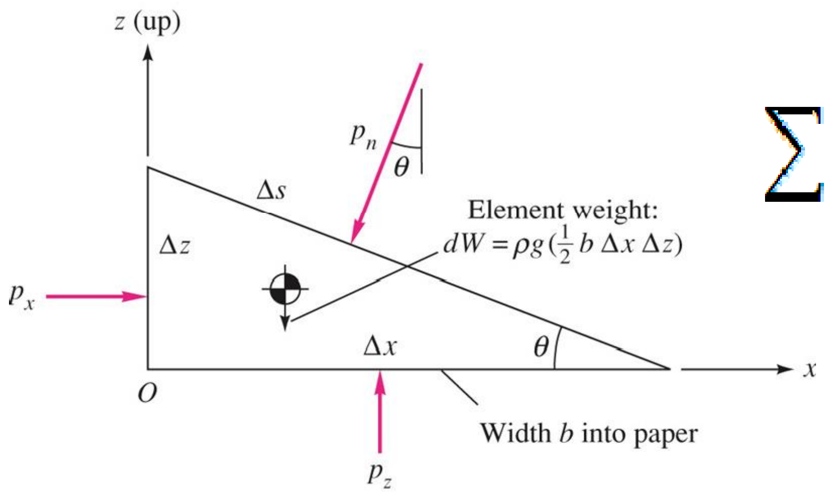
Capillary depression
Adhesion < cohesion
liquid stays away from the surface

Hydrostatics

- Pressuredistributioninastaticfluidanditseffectsonsolidsurfacesandon floating and submerged bodies.

Fluid at rest

- hydrostatic condition: when a fluid velocity is zero, the pressure variation is due only to the weight of the fluid.



$$\sum F_z = 0 = p_z\, b\, \Delta x - p_n b\, \Delta s\, cos\theta - \frac{1}{2}\rho g\, b\, \Delta x\, \Delta z$$

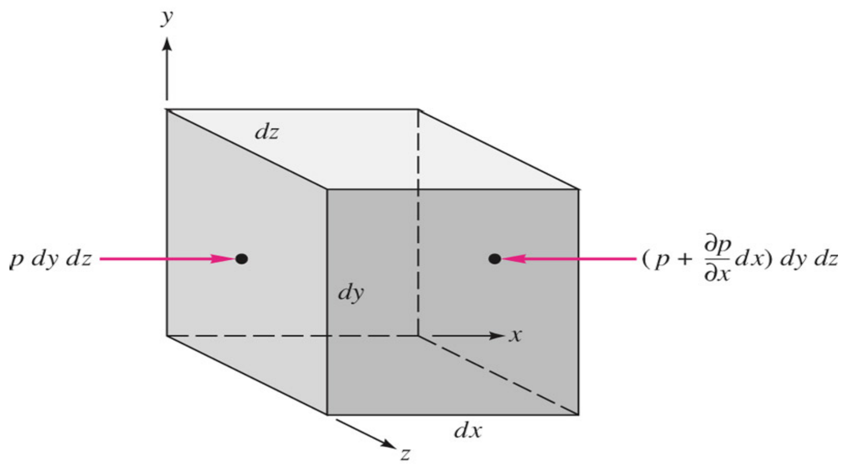
$$\sum F_x = 0 = p_x\, b\, \Delta z - p_n b\, \Delta s\, sin\theta$$

$$p_x = p_n \qquad p_z = p_n + \frac{1}{2}\rho g\, \Delta z$$

- There is no pressure change in the horizontal direction.
- There is a pressure change in the vertical direction proportional to the density, gravity, and depth change.

- In the limit when the wedge shrinks to a point, $p_x = p_z = p_n = p$

- Assumethepressurevaryarbitrarilyinafluid,$p=p(x,y,z,t)$.



$$dF_x = p\,dydz - \left(p + \frac{\partial p}{\partial x} dx\right) dydz = -\frac{\partial p}{\partial x} dx\,dy\,dz$$

$$dF_{press} = -\left(\frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k\right) dx\,dy\,dz$$

$$\boldsymbol{f}_{press} = -\nabla p$$

- The pressure gradient is a surface force that acts on the sides of the element.
- Note that the pressure gradient (not pressure) causes a net force that must be balanced by gravity or acceleration.

Equilibrium

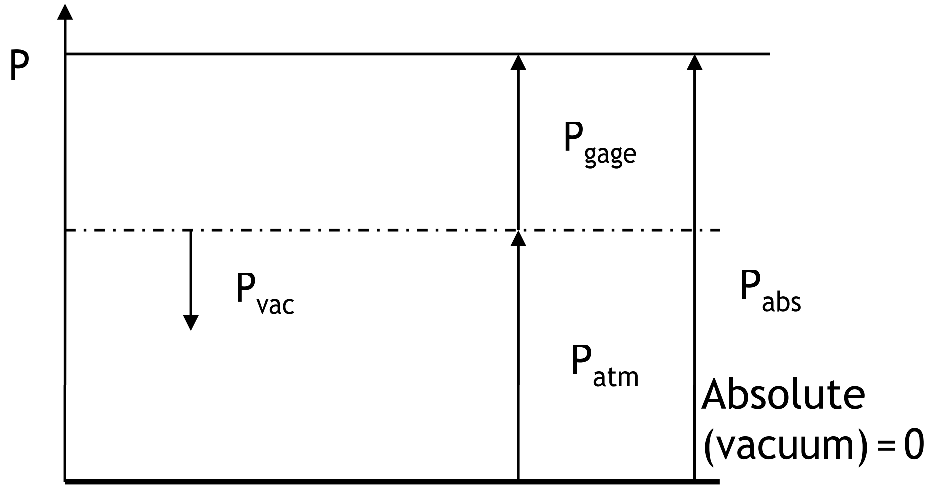
- The pressure gradient must be balanced by gravity force, or weight of the element, for a fluid at rest.

$$dF_{gravity} = \rho g dx\, dy\, dz \qquad f_{gravity} = \rho g$$

- The gravity force is a body force, acting on the entire mass of the element. Magnetic force is another example of body force.

Gage pressure and vacuum

- The actual pressure at a given position is called the <u>absolute pressure</u>, and it is measured relative to absolute vacuum.



$p > p_a$ *Gage pressure* $\quad p_{gage} = p - p_a$

$p < p_a$ *Vacuum pressure* $\quad p_{vacuum} = p_a - p$

Hydrostatic pressure distribution

- Forafluidatrest,pressuregradientmustbebalancedbythegravityforce

$$\nabla p = \rho g$$

- Recall:Apisperpendiculareverywheretosurfaceofconstantpressure*p*.
- Inourcustomarycoordinatezis“upward”andthegravityvectoris:

$$\boldsymbol{g} = -gk$$

where $g = 9.807\ m/s^2$. The pressure gradient vector becomes:

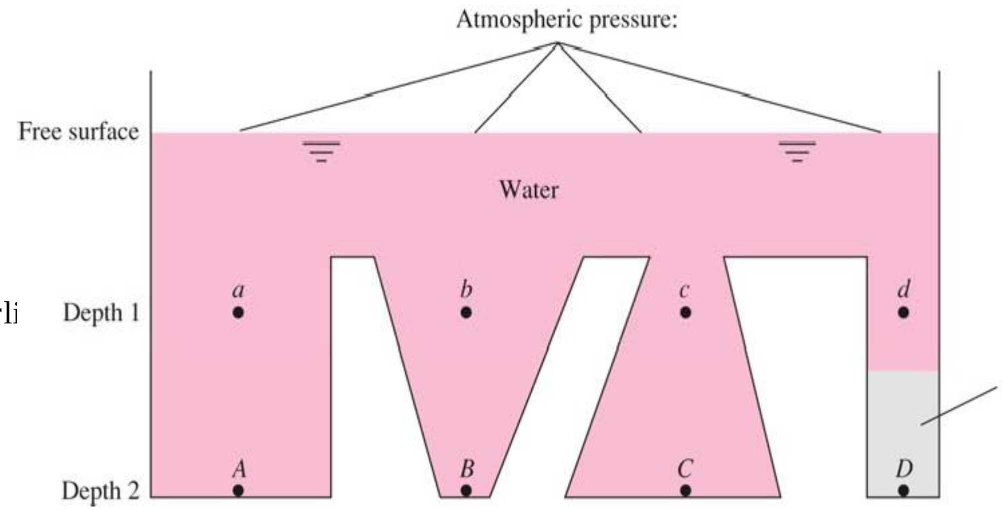
$$\frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\rho g = -\gamma$$

Hydrostatic pressure distribution

$$\frac{dp}{dz} = -\gamma \qquad p_2 - p_1 = -\int_1^2 \gamma dz$$

Hydrostatic pressure distribution

- Pressure in a continuously distributed uniform static fluid varies only with vertical distance and is independent of the shape of the container.
- The pressure is the same at all points on a given horizontal plane in a fluid.



$$p_A = p_B = p_C \neq p_D$$

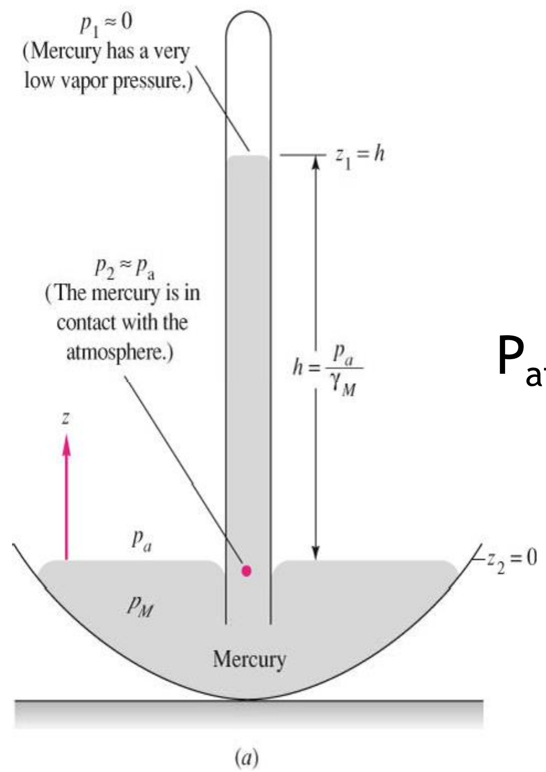
$$p_a = p_b = p_c = p_d$$

- For li

$$_2 = \frac{p_2}{\gamma} - \frac{p_1}{\gamma}$$

- The quantity, p/γ is a length called the pressure head of the fluid.

The mercury barometer



$P_{atm} = 761$ mmHg

- Mercury has an extremely small vapor pressure at room temperature (almost vacuum), thus $p_1 = 0$. One can write:

$$p_a - 0 = -\gamma_{mercury}(0 - h) \quad or \quad h = \frac{p_a}{\gamma_{mercury}}$$

Hydrostatic pressure in gases

- Gases are compressible, using the ideal gas equation of state, $p=\rho RT$:

$$\frac{dp}{dz} = -\rho g = -\frac{p}{RT} g$$

- For small variations in elevation, "isothermal atmosphere" can be assumed:

$$p_2 = p_1 exp\left[-\frac{g(z_2 - z_1)}{RT_0}\right]$$

- In general (for higher altitudes) the atmospheric temperature drops off linearly with z

$$T \approx T_0 - Bz$$

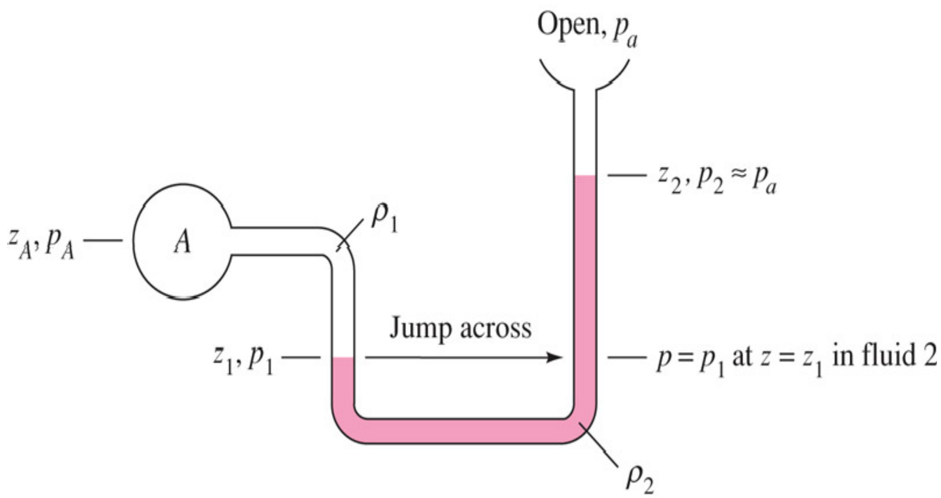
where T_0 is the sea-level temperature (in Kelvin) and B=0.00650 K/m.

$$p = p_a\left(1 - \frac{Bz}{T_0}\right)^{g/RB} \quad \text{for air } \frac{g}{RB} = 5.26$$

- Note that the P_{atm} is nearly zero (vacuum condition) at z = 30 km.

Manometry

- A static column of one or multiple fluids can be used to measure pressure difference between 2 points. Such a device is called manometer.



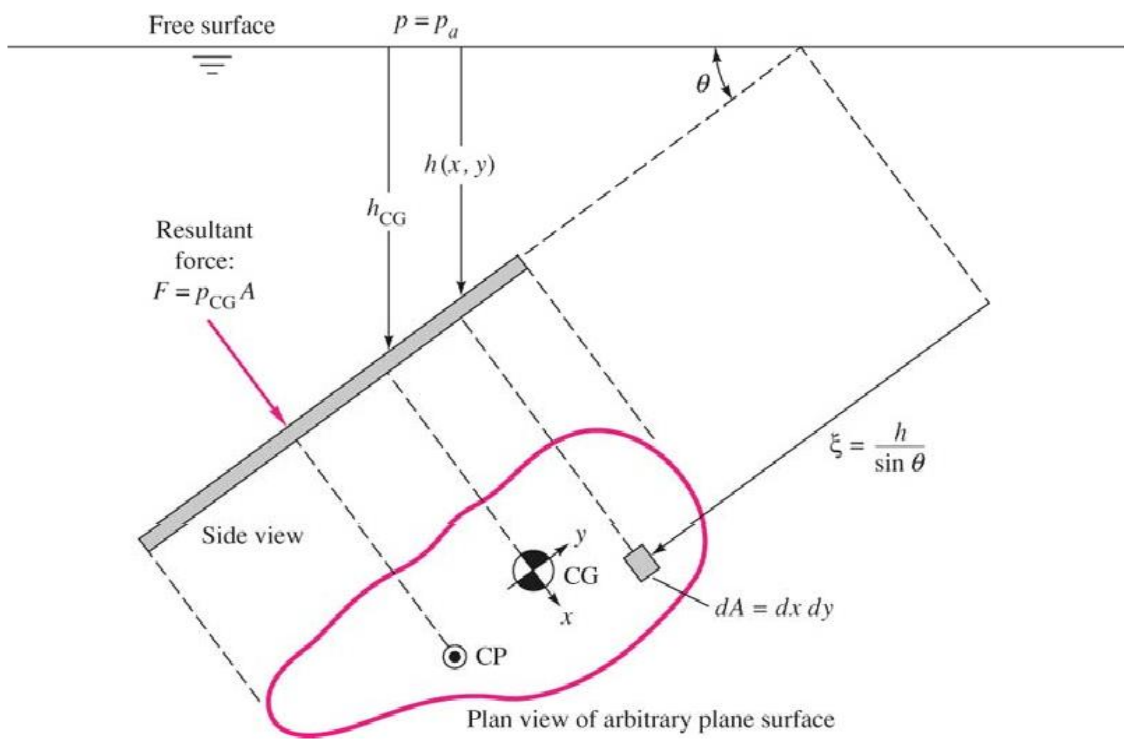
$$p_A + \gamma_1 |z_A - z_1| - \gamma_2 |z_1 - z_2| = p_2 = p_{atm}$$

- Adding/subtracting $\gamma \Delta z$ as moving down/up in a fluid column.

- Jumping across U-tubes: any two points at the same elevation in a continuous mass of the same static fluid will be at the same pressure.

Hydrostatic forces on surfaces

- Consider a plane panel of arbitrary shape completely submerged in a liquid.



$$p = p_a + \gamma h$$

- The total hydrostatic force on one side of the plane is given by:

$$F = \int p\,dA = \int (p_a + \gamma h)\,dA = p_a A + \gamma \int h\,dA$$

Hydrostatic forces on surfaces

- After integration and simplifications, we find:

$$F = p_a A + \gamma h_{CG} A = (p_a + \gamma h_{CG})A = p_{CG} A$$

- The force on one side of any plane submerged surface in a uniform fluid equals the pressure at the plate centroid times the plate area, independent of the shape of the plate or angle θ.

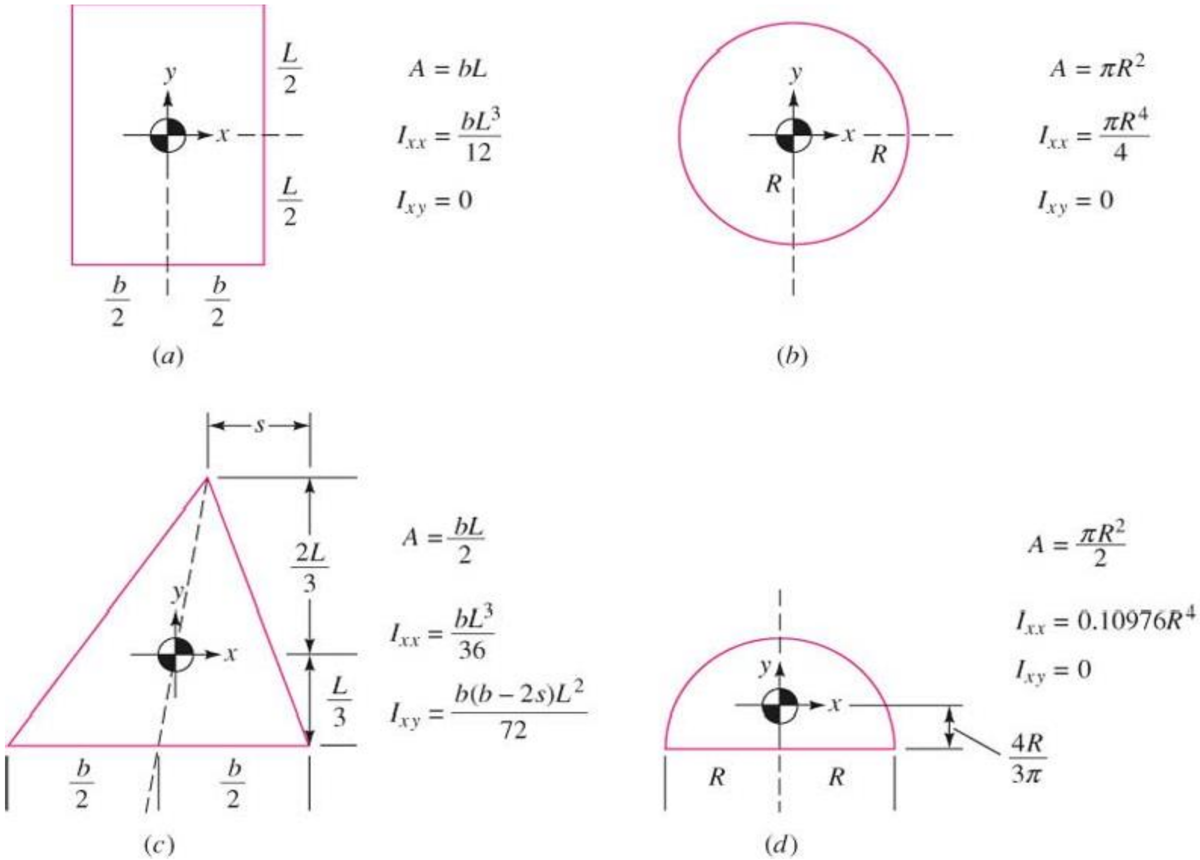
- The resultant force acts not through the centroid but below it toward the high pressure side. Its line of action passes through the centre of pressure CP of the plate (x_{CP}, y_{CP}).

$$F y_{CP} - \int y p dA - \int y(p_a + \gamma \xi sin\theta)\, dA - \gamma sin\theta \int y \xi dA$$

$$y_{CP} = -\gamma sin\theta \frac{I_{xx}}{p_{CG} A} \qquad x_{CP} = -\gamma sin\theta \frac{I_{xy}}{p_{CG} A}$$

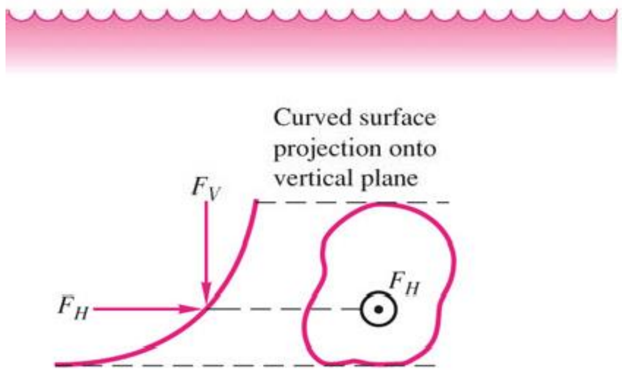
Hydrostatic forces on surfaces

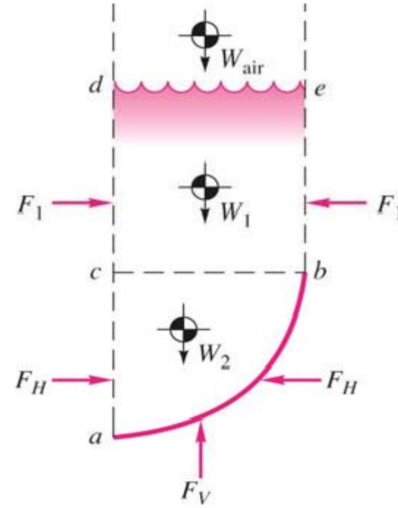
- Centroidal moments of inertia for various cross-sections.



- Note: for symmetrical plates, $I_{xy} = 0$ and thus $x_{CP} = 0$. As a result, the center of pressure lies directly below the centroid on the y axis.

- The easiest way to calculate the pressure forces on a curved surface is to compute the horizontal and vertical forces separately.





- The horizontal force equals the force on the plane area formed by the projection of the curved surface onto a vertical plane normal to the component.

- The vertical component equals to the weight of the entire column of fluid, both liquid and atmospheric above the curved surface.

$$F_V = W_2 + W_1 + W_{air}$$

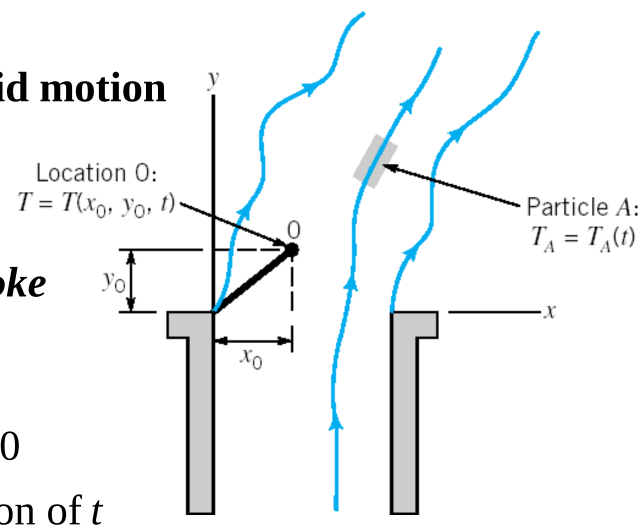
Fluid Kinematics

a) Description of motion of individual fluid molecules (X)

# of molecules per mm^3 ~ 10^{18} for gases or ~ 10^{21} for liquids

b) Description of motion of small volume of fluid (fluid particle) (O)

☐ **Two effective ways of describing fluid motion**



E.g. Smoke discharging from a chimney

Q. Determine the ***temperature (T) of smoke***

Method 1.

Step 1. Attach a thermometer at point 0

Step 2. Record T at point 0 as a function of t

$$T = T(x_0, y_0, z_0, t)$$

Step 3. Repeat the measurements at numerous points

$$T = T(x, y, z, t)$$

: Temperature information as a function of ***location***

Eulerian method (Practical)

Method 2.

Step 1. Attach a thermometer to a specific particle A

Step 2. Record T of the particle as a function of time

$$T = T_A(t)$$

Step 3. Repeat the measurements for numerous particles

$$T = T(t)$$

: Temperature information of an ***individual*** particle

Lagrangian method (Unrealistic)

- **Visualization of a flow feature**

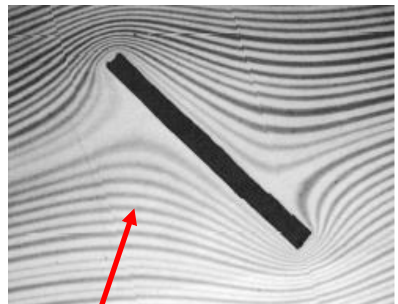
1. Streamline (Analytical purpose): ***Tangential to Velocity field***

Steady flow: Fixed lines in space and time (No shape change)

Unsteady flow: Shape changes with time

e.g. For 2-D flows, Slope of the streamlines,

$$\frac{dy}{dx} = \frac{v}{u}$$

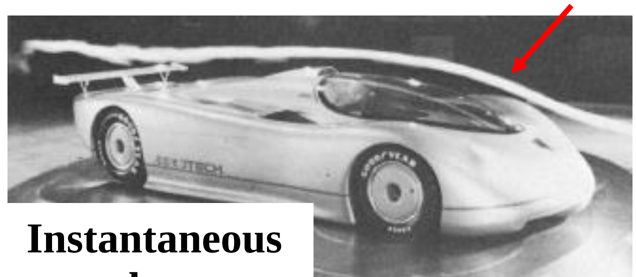


Continuous Video capture

2. ***Streakline*** (Experimental purpose): Connecting line of all particles in a flow previously passing through a common point

Steady flow: Streakline = Streamline

Unsteady flow: Different at different time



Instantaneous snapshot

3. ***Pathline*** (Experimental purpose; *Lagrangian* concept) : Traced out by a specific particle from a point to another

Steady flow: Pathline = Streamline

Unsteady flow: None of these lines need to be the same

Time exposure photograph

☐ *Eulerian* analysis vs. *Lagrangian* analysis I (Fluid Velocity)

- ***Eulerian* representation (Field representation)**

Step 1. Select a specific point (location) in space

Step 2. Measure the fluid properties (ρ, p, v , and a) at the point as functions of time

Step 3. Repeat Step 1 & 2 for numerous points (locations): ***Mapping***

∴ Results: Fluid properties (ρ, p, v , and a)
- Function of ***LOCATION*** and ***TIME*** (***Field*** representation)

e.g. Temperature in a room determined by *Eulerian* method

$T = T(x, y, z, t)$: Temperature field

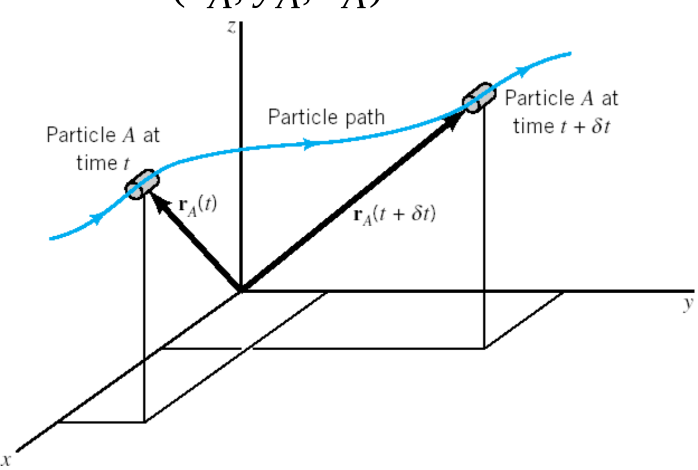
- **Velocity Field of Fluid flow**
  - Velocity information as a function of location and time

$$V = u(x, y, z, t)\hat{i} + v(x, y, z, t)\hat{j} + w(x, y, z, t)\hat{k}$$

where ***u, v, w*** : *x, y, z* components of *V* at (*x*,*y*,*z*) and time *t*
(Velocity distribution in space at certain time *t*)

※ How to determine Velocity of a specific particle *A* at time *t*,
- ***Must know*** Location of particle *A* = (x_A, y_A, z_A) at time *t*

$$\vec{V}_A = u(x_A, y_A, z_A, t)\hat{i} + v(x_A, y_A, z_A, t)\hat{j} + w(x_A, y_A, z_A, t)\hat{k}$$



☐ Additional conditions

- **Steady and Unsteady Flows**

a) **Steady flow** (Time-independent flowing feature)

$$V = u(x, y, z)\hat{i} + v(x, y, z)\hat{j} + w(x, y, z)\hat{k}$$

: Velocity field doesn't vary with time.

b) **Unsteady flow** (Time-dependent flowing feature)

$$V = u(x, y, z, t)\hat{i} + v(x, y, z, t)\hat{j} + w(x, y, z, t)\hat{k}$$

Type 1. Nonperiodic, unsteady flow:

e.g. Turn off the faucet to stop the water flow.

Type 2. Periodic, unsteady flow

e.g. Periodic injection of air-gasoline mixture into the cylinder of an automobile engine.

Type 3. : Pure random, unsteady flow: Turbulent flow

c.f. ***Lagrangian*** description of the fluid velocity

$$\vec{v}_A = u_A(t)\hat{i} + v_A(t)\hat{j} + w_A(t)\hat{k}$$ for individual particles *A*

☐ *Eulerian* analysis vs. *Lagrangian* analysis II (Fluid Acceleration)

Eularian method: ***Acceleration field*** (function of location and time)

Langrangian method: $a_A = a_A(t)$ for individual particles *A*

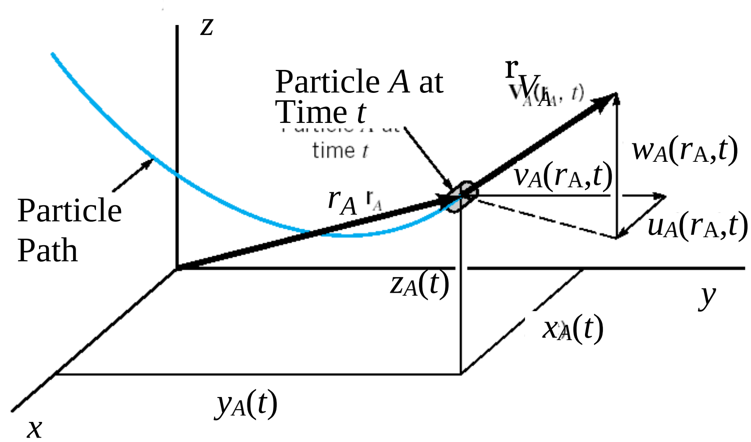
Consider a fluid particle *A* moving along its ***pathline***

From Velocity field,

$$\vec{V} = \vec{V}(x, y, z, t)$$

Velocity $\vec{V}_A$ for particle *A*,

$$V_A = V_A\ [x_A(t), y_A(t), z_A(t), t]$$



Acceleration a_A of particle *A*

$$\vec{a}_A(t) = \frac{dV_A}{dt} = \frac{\partial V_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial V_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial V_A}{\partial z}\frac{dz_A}{dt} + \frac{\partial V_A}{\partial t}$$

From the velocity field $\rightarrow$ $u_A = \frac{dx_A}{dt},\ v_A = \frac{dy_A}{dt},\ w_A = \frac{dz_A}{dt}$

$$\vec{a}_A(t) = \frac{dV_A}{dt} = u_A\frac{\partial V_A}{\partial x} + v_A\frac{\partial V_A}{\partial y} + w_A\frac{\partial V_A}{\partial z} + \frac{\partial V_A}{\partial t}$$

By applying this equation to all particles in a flow at the same time,

$$\therefore\ \vec{a}(t) = \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z} \qquad \text{(Vector equation)}$$

x-component $a_x(t) = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$

y-component $a_y(t) = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$

z-component $a_z(t) = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$

- Simple representation of the equation (***Material Derivative***)

$$\vec{a}(t) = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} \equiv \frac{D\vec{V}}{Dt}$$

where $\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$ or

$$= \frac{\partial(\ )}{\partial t} + (\vec{V}\cdot\nabla)(\ )$$

: Material derivative or Substantial derivative

※ ***Material derivative***: Time rate of change of fluid properties

- Related with both ***Time-dependent change*** and ***Fluid's motion*** (Velocity field (or u, v, w): Must be known]

e.g. Time rate of change of ***temperature***

$$\frac{dT_A}{dt} = \frac{\partial T_A}{\partial t} + \frac{\partial T_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial T_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial T_A}{\partial z}\frac{dz_A}{dt} \quad \text{(For a particle } A\text{)}$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\partial T}{\partial t} + V\cdot\nabla T \quad \text{(For any particle)}$$

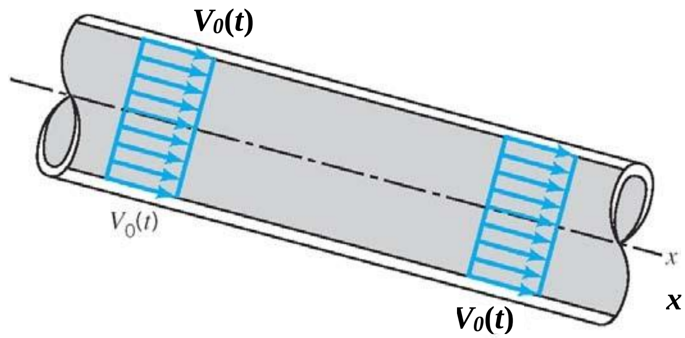
☐ Relation between Material Derivative and Steadiness of the flow

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$$

***Local* derivative** due to unsteady effect

e.g. **Uniform flow**

Consider the situation shown



$V = V_0\,(t)\hat{i}$ (Spatially uniform)

Then the acceleration field,

$$a = \frac{\partial V}{\partial t} + u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z} = \frac{\partial V}{\partial t} = \frac{\partial V_0}{\partial t}\hat{i}$$

: Uniform, but not necessarily constant in time

☐ Relation between Material Derivative and the Fluid Motion

$$\frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z}$$

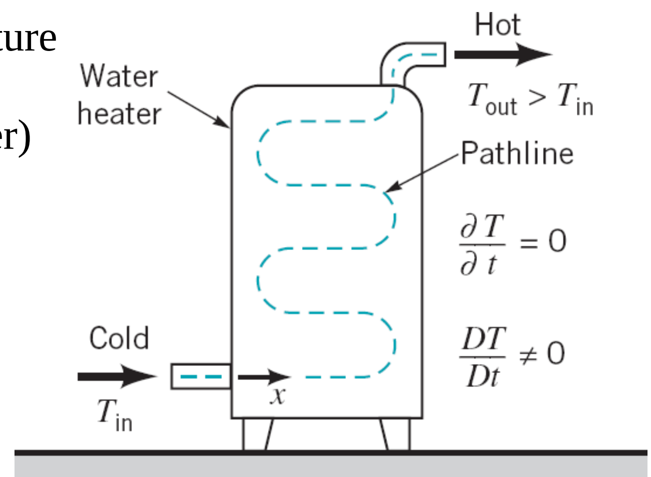
***Spatial* (or *Convective*) derivative**
- Variation due to the motion of fluid particle

- ***Convective*** acceleration = $(V\cdot\nabla)V$: Due to the convection (or motion) of the particle from one point to another point

- Convective effect: Regardless of steady or unsteady flow

e.g. 1. Convective effect on the temperature

Consider a situation shown (Water heater)



T_{in} (entering): Constant low temp.

T_{out} (leaving): Constant high temp.

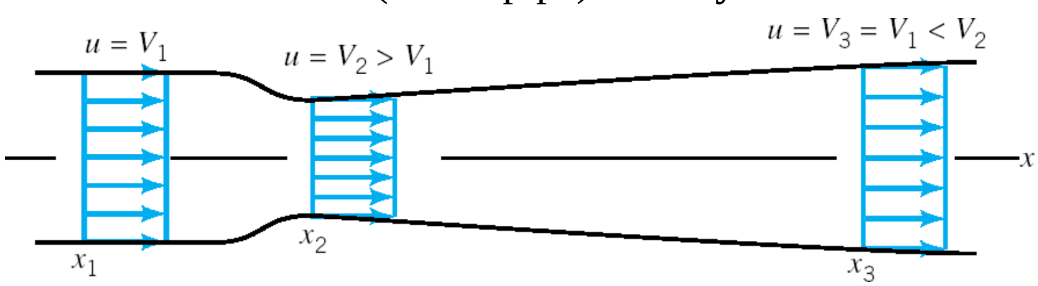
: Steady flow

But, $\frac{DT}{Dt} = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} \neq 0$

➲ Temperature of each particle: Increase from inlet to outlet

e.g. 2. Convective effect on the acceleration

Consider a situation shown (Water pipe): **Steady flow**



(1) → (2) ($x_1 < x < x_2$)

Velocity increases ($V_1 < V_2$): $\frac{\partial V}{\partial t} = 0$, but acceleration $a_x = u\frac{\partial u}{\partial x} > 0$

(2) → (3) ($x_2 < x < x_3$)

Velocity decreases ($V_2 > V_3$): $\frac{\partial V}{\partial t} = 0$, but acceleration $a_x = u\frac{\partial u}{\partial x} < 0$

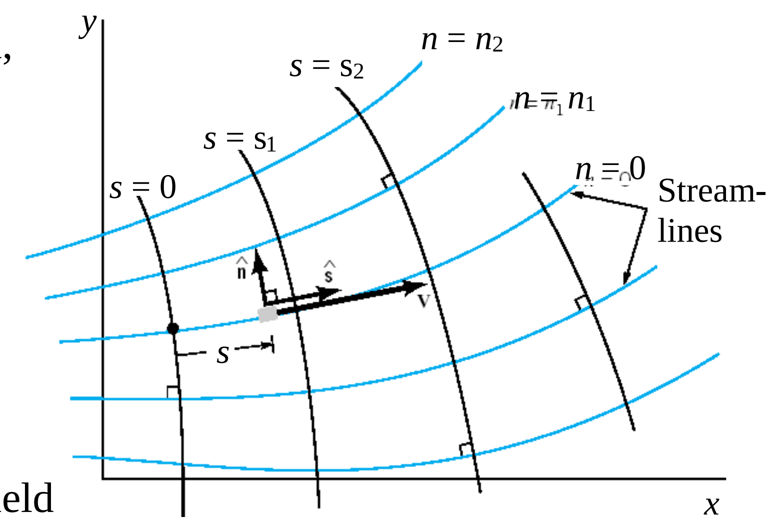
: Due to the convective acceleration

☐ Streamline coordinates again (Easy to describe the fluid motion)

Consider 2-D steady flow shown,

a) Cartesian coordinates: x, y
: Unit vectors $\hat{i}$, $\hat{j}$

b) Streamline coordinates: s, n
: Unit vectors $\hat{s}$, $\hat{n}$



From the definition of velocity field

$V = V\hat{s}$ (always tangent to the streamline direction)

Thus, for steady 2D flow,

$$\vec{a} = \frac{DV}{Dt} = \frac{DV\hat{s}}{Dt} = \frac{DV}{Dt}\hat{s} + V\frac{D\hat{s}}{Dt} \quad \text{(By the chain rule)}$$

$$= a_s\,\hat{s} + a_n\,\hat{n}$$

Then,

0: along streamline | 0: along streamline

$$a = \left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial n}\frac{dn}{dt}\right)\hat{s} + V\left(\frac{\partial \hat{s}}{\partial t} + \frac{\partial \hat{s}}{\partial s}\frac{ds}{dt} + \frac{\partial \hat{s}}{\partial n}\frac{dn}{dt}\right)$$

0: Steady flow | 0: Steady flow

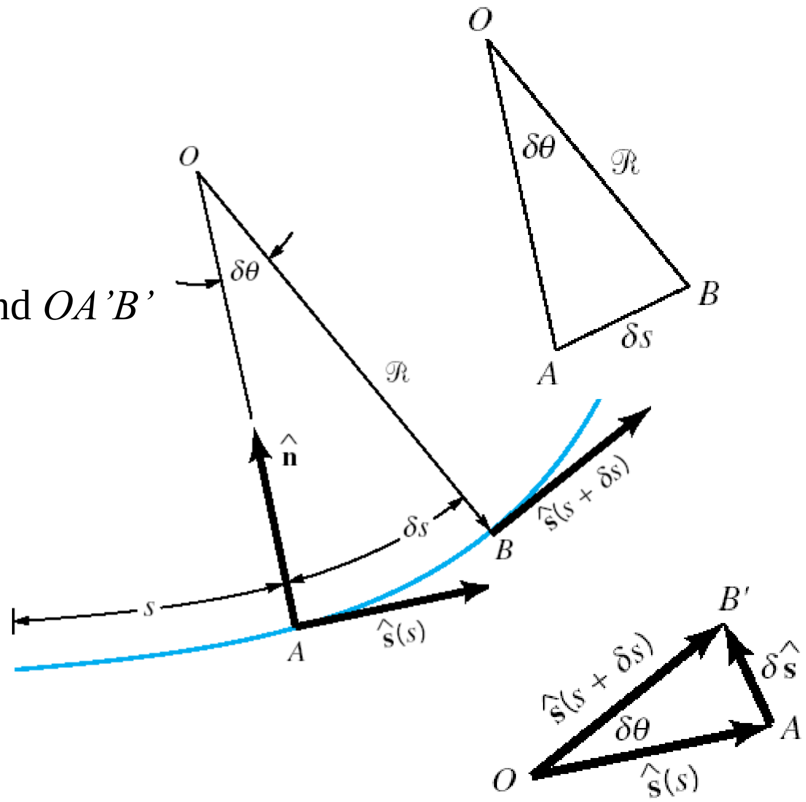
$$a = \left(\frac{\partial V}{\partial s}\frac{ds}{dt}\right)\hat{s} + V\left(\frac{\partial \hat{s}}{\partial s}\frac{ds}{dt}\right) = \left(V\frac{\partial V}{\partial s}\right)\hat{s} + VV\left(\frac{\partial \hat{s}}{\partial s}\right)$$

where $\frac{ds}{dt} = V$ and $\frac{\partial \hat{s}}{\partial s}$: Change in direction ($\hat{s}$) per δs

As seen in the Figure,

$$\frac{\delta s}{R} = \frac{|\delta \hat{s}|}{|\hat{s}|} = |\delta \hat{s}|$$

: Comparing *OAB* and *OA'B'*



Thus,

$$\frac{\partial \hat{s}}{\partial s} = \lim_{\delta s \to 0} \frac{\delta \hat{s}}{\delta s} = \frac{\hat{n}}{R}$$

Finally,

$$\therefore \vec{a} = V\frac{\partial V}{\partial s}\hat{s} + \frac{V^2}{R}\hat{n} \qquad \text{or} \qquad a_s = V\frac{\partial V}{\partial s}, \qquad a_n = \frac{V^2}{R}$$

$a_s = V\dfrac{\partial V}{\partial s}$: Convective accel. along the streamline (change in speed)

$a_n = \dfrac{V^2}{R}$: Centripetal accel. normal to the streamline(change in direction)

※***Same as those in the previous chapter***

☐ Defining Volume of fluid in motion

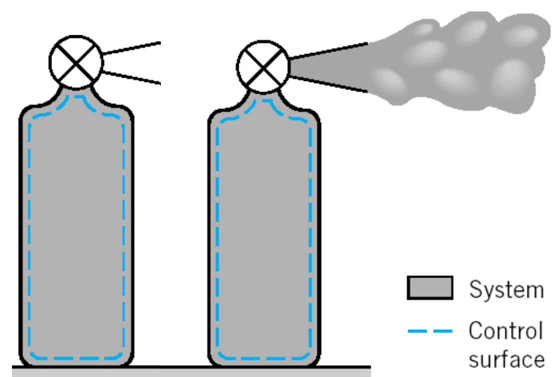
※ Almost analysis related with fluid mechanics

- Focus on ***Motion (or interaction) of a specific amount of fluid*** or ***Motion of fluid in a specific volume***

- Two typical boundaries of fluid of interest

a) **System**: *Lagrangian* concept (Focus on **real material**)

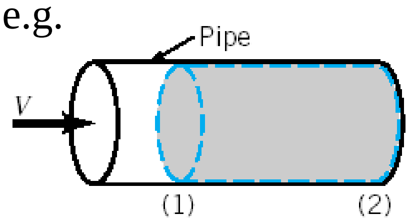
- Specific quantity of identified (tagged) fluid matter
- A variety of interactions with surrounding (Heat transfer, Exertion of the pressure force)
- Possibly change in size and shape
- Always constant mass (Conservation)



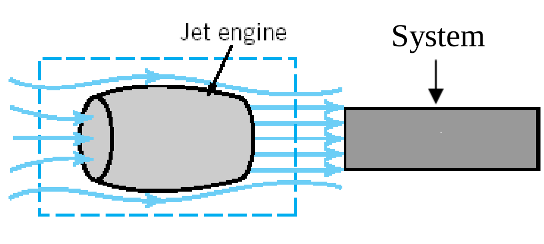
b) **Control volume**: *Eulerian* concept (Focus on fluid in **specific location**)

- Specific geometric volume in space
- Interested in the fluid within the volume
- Amount (Mass) within the volume: Change with time

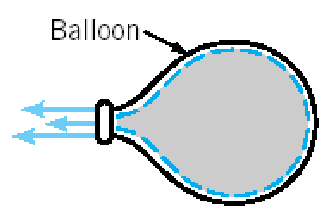
e.g.



Fixed



Fixed or moving



Deforming

※ Mostly in this textbook, only ***fixed, uondeformable control volume*** will be considered.

Fluid Dynamics

FUNDAMENTAL LAWS AND EQUATIONS

Kinematics

What is a fluid? Specification of motion

A fluid is anything that flows, usually a liquid or a gas, the latter being distinguished by its great relative compressibility.

Fluids are treated as continuous media, and their motion and state can be specified in terms of the velocity $\mathbf{u}$, pressure p, density ρ, etc evaluated at every point in space $\mathbf{x}$ and time t. To define the density at a point, for example, suppose the point to be surrounded by a very small element (small compared with length scales of interest in experiments) which nevertheless contains a very large number of molecules. The density is then the total mass of all the molecules in the element divided by the volume of the element.

Considering the velocity, pressure, etc as functions of time and position in space is consistent with measurement techniques using fixed instruments in moving fluids. It is called the *Eulerian specification*. However, Newton's laws of motion (see below) are expressed in terms of individual particles, or fluid elements, which move about. Specifying a fluid motion in terms of the position $\mathbf{X}(t)$ of an individual particle (identified by its initial position, say) is called the *Lagrangian specification*. The two are linked by the fact that the velocity of such an element is equal to the velocity of the fluid evaluated at the position occupied by the element:

$$\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t} = \mathbf{u}\left[\mathbf{X}(t), t\right] . \quad (1)$$

The path followed by a fluid element is called a *particle path*, while a curve which, at any instant, is everywhere parallel to the local fluid velocity vector

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is called a streamline. Particle paths are coincident with streamlines in steady flows, for which the velocity **u** at any fixed point **x** does not vary with time t.

Material derivative; acceleration.

Newton's Laws refer to the acceleration of a particle. A fluid element may have acceleration both because the velocity at its location in space is changing (*local acceleration*) and because it is moving to a location where the velocity is different (*convective acceleration*). The latter exists even in a steady flow.

How to evaluate the rate of change of a quantity at a moving fluid element, in the Eulerian specification? Consider a scalar such as density $\rho(\mathbf{x},t)$. Let the particle be at position **x** at time t, and move to $\mathbf{x} + \delta\mathbf{x}$ at time $t + \delta t$, where (in the limit of small δt)

$$\delta\mathbf{x} = \mathbf{u}(\mathbf{x},t)\delta t \ . \qquad (2)$$

Then the rate of change of ρ following the fluid, or *material derivative*, is

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = \lim_{\delta t\to 0} \frac{\rho(\mathbf{x}+\delta\mathbf{x}, t+\delta t) - \rho(\mathbf{x},t)}{\delta t}$$

$$= \frac{\partial\rho}{\partial x}\frac{\delta x}{\delta t} + \frac{\partial\rho}{\partial y}\frac{\delta y}{\delta t} + \frac{\partial\rho}{\partial z}\frac{\delta z}{\delta t} + \frac{\partial\rho}{\partial t}$$

(by the chain rule for partial differentiation)

$$= \frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} + w\frac{\partial\rho}{\partial z} \qquad (3a)$$

(using (2))

$$= \frac{\partial\rho}{\partial t} + \mathbf{u}.\nabla\rho \qquad (3b)$$

in vector notation, where the vector $\nabla\rho$ is the gradient of the scalar field ρ:

$$\nabla\rho = \left(\frac{\partial\rho}{\partial x}, \frac{\partial\rho}{\partial y}, \frac{\partial\rho}{\partial z}\right).$$

A similar exercise can be performed for each component of velocity, and we can write the x-component of acceleration as

$$\frac{\mathrm{D}u}{\mathrm{D}t} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}, \qquad (4a)$$

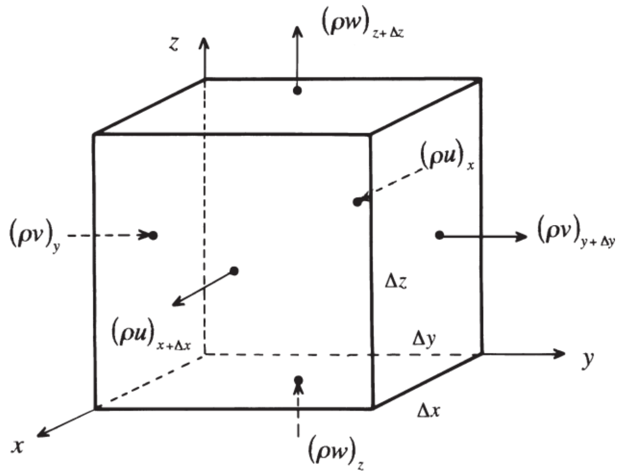

FIG. 1. – Mass flow into and out of a small rectangular region of space.

etc. Combining all three components in vector shorthand we write

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \frac{\partial\mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u}, \qquad (4b)$$

but care is needed because the quantity $\nabla\mathbf{u}$ is not defined in standard vector notation. Note that $\partial\mathbf{u}/\partial t$ is the local acceleration, $(\mathbf{u}.\nabla)\mathbf{u}$ the convective acceleration. Note too that the convective acceleration is *nonlinear* in **u**, which is the source of the great complexity of the mathematics and physics of fluid motion.

Conservation of mass

This is a fundamental principle, stating that for any closed volume fixed in space, the rate of increase of mass within the volume is equal to the net rate at which fluid enters across the surface of the volume. When applied to the arbitrary small rectangular volume depicted in fig. 1, this principle gives:

$$\Delta x\Delta y\Delta z\frac{\partial\rho}{\partial t} = \Delta y\Delta z\left([\rho u]_x - [\rho u]_{x+\Delta x}\right) +$$

$$+\Delta z\Delta x\left([\rho v]_y - [\rho v]_{y+\Delta y}\right) +$$

$$+\Delta x\Delta y\left([\rho w]_z - [\rho w]_{z+\Delta z}\right).$$

Dividing by $\Delta x\ \Delta y\ \Delta z$ and taking the limit as the volume becomes very small we get

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho u) - \frac{\partial}{\partial y}(\rho v) - \frac{\partial}{\partial z}(\rho w) \qquad (5a)$$

or (in shorthand)

$$\frac{\partial \rho}{\partial t} = -\mathrm{div}(\rho \mathbf{u}) \qquad (5b)$$

where we have introduced the *divergence* of a vector. Differentiating the products in (5a) and using (3), we obtain

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = -\rho \mathrm{div}\mathbf{u}. \qquad (6)$$

This says that the rate of change of density of a fluid element is positive if the divergence of the velocity field is negative, i.e. if there is a tendency for the flow to converge on that element.

If a fluid is *incompressible* (as liquids often are, effectively) then even if its density is not uniform everywhere (e.g. in a stratified ocean) the density of each fluid element cannot change, so

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = 0 \qquad (7)$$

everywhere, and the velocity field *must* satisfy

$$\mathrm{div}\mathbf{u} = \mathbf{0} \qquad (8a)$$

or

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \,. \qquad (8b)$$

This is an important constraint on the flow of an incompressible fluid.

The Navier-Stokes equations

Newton's Laws of Motion

Newton's first two laws state that if a particle (or fluid element) has an acceleration then it must be experiencing a force (vector) equal to the product of the acceleration and the mass of the particle:

force = *mass* × *acceleration*.

For any collection of particles this becomes

net force = rate of change of momentum

where the momentum of a particle is the product of its mass and its velocity. Newton's third law states that, if two elements A and B exert forces on each other, the force exerted by A on B is the negative of the force exerted by B on A.

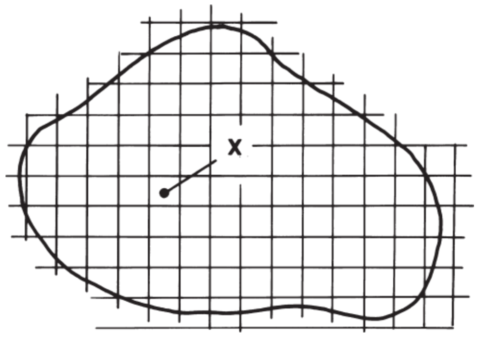

Fig. 2. – An arbitrary region of fluid divided up into small rectangular elements (depicted only in two dimensions).

To apply these laws to a region of continuous fluid, the region must be thought of as split up into a large number of small fluid elements (fig. 2), one of which, at point **x** and time t, has volume ΔV, say. Then the mass of the element is $\rho(\mathbf{x},t)\,\Delta V$, and its acceleration is $\mathrm{D}\mathbf{u}/\mathrm{D}t$ evaluated at $(\mathbf{x},t)$. What is the force?

Body force and stress

The force on an element consists in general of two parts, a *body force* such as gravity exerted on the element independently of its neighbours, and *surface forces* exerted on the element by all the other elements (or boundaries) with which it is in contact. The gravitational body force on the element ΔV is $\mathbf{g}\rho(\mathbf{x}, t)\,\Delta V$, where **g** is the gravitational acceleration. The surface force acting on a small planar surface, part of the surface of the element of interest, can be shown to be proportional to the area of the surface, ΔA say, and simply related to its orientation, as represented by the perpendicular (normal) unit vector **n** (fig. 3). The force per unit area, or *stress*, is then given by

Fig. 3. – Surface force on an arbitrary small surface element embedded in the fluid, with area ΔA and normal **n**. **F** is the force exerted *by* the fluid on side 1, *on* the fluid on side 2.

$$F_x = \sigma_{xx} n_x + \sigma_{xy} n_y + \sigma_{xz} n_z$$
$$F_y = \sigma_{yx} n_x + \sigma_{yy} n_y + \sigma_{yz} n_z \qquad (9a)$$
$$F_z = \sigma_{zx} n_x + \sigma_{zy} n_y + \sigma_{zz} n_z$$

or, in shorthand,

$$\mathrm{F} = \underset{\approx}{\sigma}\, \mathbf{n} \qquad (9b)$$

where $\underset{\approx}{\sigma}$ is a matrix quantity, or *tensor*, depending on $\mathbf{x}$ and t but not $\mathbf{n}$ or ΔA. $\underset{\approx}{\sigma}$ is called the *stress tensor*, and can be shown to be *symmetric* (i.e. $\sigma_{yx} = \sigma_{xy}$, etc) so it has just 6 independent components.

It is an experimental observation that the *stress in a fluid at rest* has a magnitude independent of $\mathbf{n}$ and is always parallel to $\mathbf{n}$ and negative, i.e. compressive. This means that $\sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0$, $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -p$, say, where p is the positive *pressure* (hydrostatic pressure); alternatively,

$$\underset{\approx}{\sigma} = -p\, \underset{\approx}{\mathbf{I}} \qquad (10)$$

where $\underset{\approx}{\mathbf{I}}$ is the identity matrix.

The relation between stress and deformation rate

In a moving fluid, the motion of a general fluid element can be thought of as being broken up into three parts: translation as a rigid body, rotation as a rigid body, and deformation (see fig. 4). Quantitatively, the translation is represented by the velocity field $\mathbf{u}$, the rigid rotation is represented by the curl of the velocity field, or *vorticity*,

$$\omega = \mathrm{curl}\,\mathbf{u}\,, \qquad (11)$$

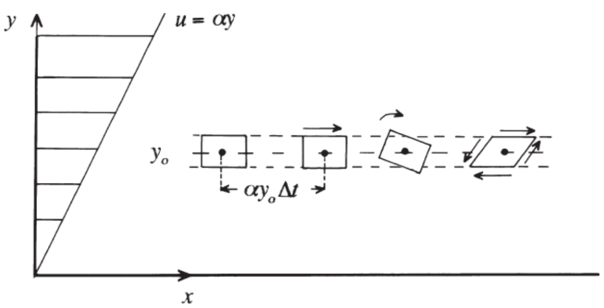

FIG. 4. – A unidirectional shear flow in which the velocity is in the x- direction and varies linearly with the perpendicular component y : $u = \alpha y$. In time Δt a small rectangular fluid element at level y_0 is *translated* a distance $\alpha y_0 \Delta t$, *rotated* through an angle $\alpha/2$, and *deformed* so that the horizontal surfaces remain horizontal, and the vertical surfaces are rotated through an angle α.

and the deformation is represented by the *rate of deformation* (or rate of strain) $\underset{\approx}{\mathbf{e}}$ which, like stress, is a symmetric tensor quantity made up of the symmetric part of the velocity gradient tensor. Formally,

$$\underset{\approx}{\mathbf{e}} = \frac{1}{2}\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right) \qquad (12)$$

or, in full component form,

$$\underset{\approx}{\mathbf{e}} = \begin{pmatrix} \dfrac{\partial u}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) & \dfrac{1}{2}\left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) & \dfrac{\partial v}{\partial y} & \dfrac{1}{2}\left(\dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x} + \dfrac{\partial u}{\partial z}\right) & \dfrac{1}{2}\left(\dfrac{\partial w}{\partial y} + \dfrac{\partial v}{\partial z}\right) & \dfrac{\partial w}{\partial z} \end{pmatrix} \qquad (13)$$

Note that the sum of the diagonal elements of $\underset{\approx}{\mathbf{e}}$ is equal to div $\mathbf{u}$.

It is a further matter of experimental observation that, whenever there is motion in which deformation is taking place, a stress is set up in the fluid which tends to resist that deformation, analogous to friction. The property of the fluid that causes this stress is its viscosity. Leaving aside pathological ('non-Newtonian') fluids the resisting stress is generally proportional to the deformation rate. Combining this stress with pressure, we obtain the *constitutive equation for a Newtonian fluid*:

$$\underset{\approx}{\sigma} = -p\,\underset{\approx}{\mathbf{I}} + 2\mu\,\underset{\approx}{\mathbf{e}} - 2/3\mu\ \mathrm{div}\,\mathbf{u}\underset{\approx}{\mathbf{I}} \qquad (14)$$

The last term is zero in an incompressible fluid, and we shall ignore it henceforth. The quantity μ is the dynamic viscosity of the fluid.

To illustrate the concept of viscosity, consider the unidirectional shear flow depicted in fig. 4 where the plane y=0 is taken to be a rigid boundary. The normal vector $\mathbf{n}$ is in the y-direction, so equations (9) show that the stress on the boundary is

$$\mathbf{F} = \left(\sigma_{xy}, \sigma_{yy}, \sigma_{zy}\right).$$

From (14) this becomes

$$\mathbf{F} = \left(2\mu e_{xy}, -p + \mu e_{yy}, \mu e_{zy}\right),$$

but because the velocity is in the x-direction only and varies with y only, the only non-zero component of $\underset{\approx}{\mathbf{e}}$ is $e_{xy} = \frac{1}{2}\frac{\partial u}{\partial y}$. Hence

$$\mathbf{F} = \left(\mu\frac{\partial u}{\partial y}, -p, 0\right)$$

In other words, the boundary experiences a perpendicular stress, downwards, of magnitude p, the pressure, and a tangential stress, in the x-direction, equal to μ times the velocity gradient $\partial u/\partial y$. (It can be seen from (9) and (14) that tangential stresses are always of viscous origin.)

The Navier-Stokes equations

The easiest way to apply Newton's Laws to a moving fluid is to consider the rectangular block element in fig. 5. Newton's Law says that the mass of the element multiplied by its acceleration is equal to the total force acting on it, i.e. the sum of the body force and the surface forces over all six faces. The resulting equation is a vector equation; we will consider just the x-component in detail. The x-component of the stress forces on the faces perpendicular to the x-axis is the difference between the perpendicular stress σ_{xx} evaluated at the right-hand face ($x+\Delta x$) and that evaluated at the left-hand face (x) multiplied by the area of those faces, $\Delta y\Delta z$, i.e.

$$\left(\sigma_{xx}\big|_{x+\Delta x} - \sigma_{xx}\big|_{x}\right)\Delta y\Delta z.$$

If Δx is small enough, this is

$$\frac{\partial \sigma_{xx}}{\partial x}\Delta x\Delta y\Delta z.$$

The x-component of the forces on the faces perpendicular to the y-axis is

$$\left(\sigma_{xy}\big|_{y+\Delta y} - \sigma_{xy}\big|_{y}\right)\Delta z\Delta x = \frac{\partial \sigma_{xy}}{\partial y}\Delta x\Delta y\Delta z,$$

and similarly for the faces perpendicular to the z-axis. Hence the x-component of Newton's Law gives

$$(\rho\Delta x\Delta y\Delta z)\frac{\mathrm{D}u}{\mathrm{D}t} = (\rho g_x)\Delta x\Delta y\Delta z + \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}\right)\Delta x\Delta y\Delta z$$

or, dividing by the element volume,

$$\rho\frac{\mathrm{D}u}{\mathrm{D}t} = \rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z}. \qquad (15a)$$

Similar equations arise for the y- and z-components, and they can be combined in vector form to give

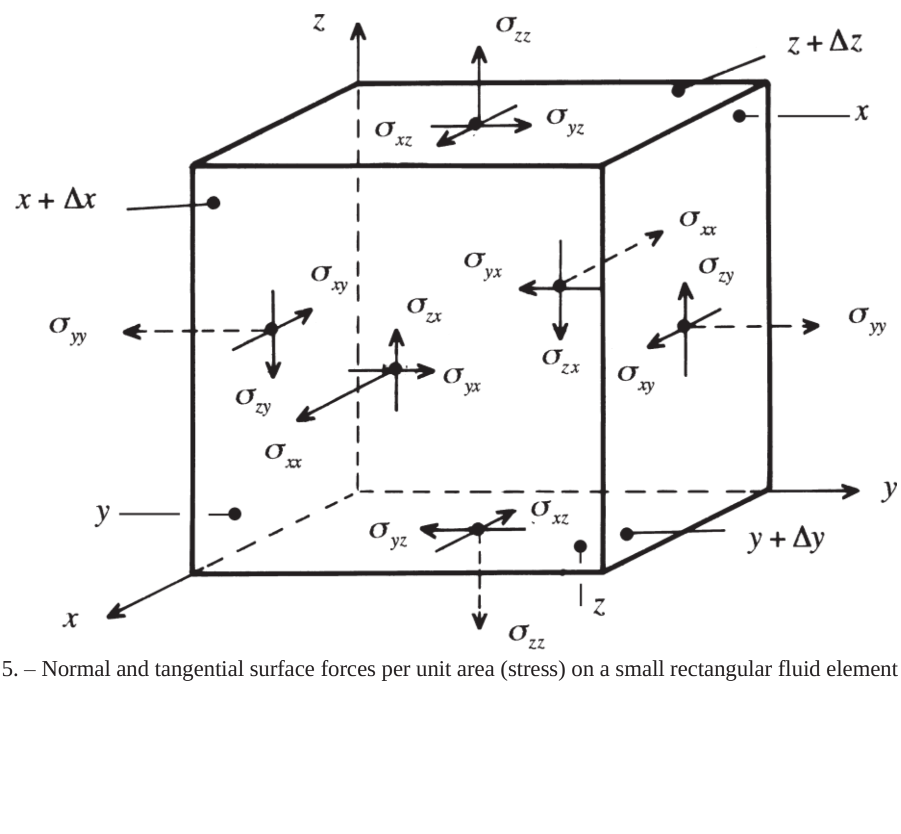

Fig. 5. – Normal and tangential surface forces per unit area (stress) on a small rectangular fluid element in motion.

$$\rho \frac{D\mathbf{u}}{Dt} = \rho \mathbf{g} + div\, \underset{\approx}{\sigma} \qquad (15b)$$

The equations can be further transformed, using the constitutive equation (14) (with div **u** = 0) and (13) to express $\underset{\approx}{\mathbf{e}}$ in terms of **u**, to give for (15a)

$$\rho \frac{Du}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right). \qquad (16a)$$

Similarly in the y- and z-directions:

$$\rho \frac{Dv}{Dt} = \rho g_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \qquad (16b)$$

$$\rho \frac{Dw}{Dt} = \rho g_z - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right). \qquad (16c)$$

In these equations, it should not be forgotten that Du/Dt etc are given by equations (4).

Finally, the above three equations can be compressed into a single vector equation as follows:

$$\rho \frac{D\mathbf{u}}{Dt} = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{u} \qquad (16d)$$

where the symbol ∇^2 is shorthand for

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial z^2}.$$

Equations (16a-c), or (16d), are the Navier-Stokes equations for the motion of a Newtonian viscous fluid. Recall that the left side of (16d) represents the mass-acceleration, or *inertia* terms in the equation, while the three terms on the right side are respectively the *body force*, the *pressure gradient*, and the *viscous term*.

The four equations (16a-c) and (8b) are four non-linear partial differential equations governing four unknowns, the three velocity components u,v,w, and the pressure p, each of which is in general a function of four variables, x, y, z and t. Note that if the density ρ is variable, that is a fifth unknown, and the corresponding fifth equation is (7). Not surprisingly, such equations cannot be solved in general, but they can be used as a framework to understand the physics of fluid motion in a variety of circumstances.

A particular simplification that can sometimes be made is to neglect viscosity altogether (to assume that the fluid is *inviscid*). Conditions in which this is permitted are discussed below. When it is allowed, however, we can put $\mu = 0$ in equations (16) and these are greatly simplified.

For quantitative purposes we should note the values of density and viscosity for fresh water and air at 1 atmosphere pressure and at different temperatures:

Temp	Water $\rho(kgm^{-3})$	Water $\mu(kgm^{-1}s^{-1})$	Air (dry) $\rho(kgm^{-3})$	Air (dry) $\mu(kgm^{-1}s^{-1})$
0˚C	1.0000×10^3	1.787×10^{-3}	1.293	1.71×10^{-5}
10˚C	0.9997×10^3	1.304×10^{-3}	1.247	1.76×10^{-5}
20˚C	0.9982×10^3	1.002×10^{-3}	1.205	1.81×10^{-5}

Boundary conditions

Whether the fluid is viscous or not, it cannot cross the interface between itself and another medium (fluid or solid), so the normal component of velocity of the fluid at the interface must equal the normal component of the velocity of the interface itself:

$$u_n = U_n \text{ or } \mathbf{n.u} = \mathbf{n.U} \qquad (17a)$$

where **U** is the interface velocity. In particular, on a solid boundary at rest,

$$\mathbf{n.u} = 0 \qquad (17b)$$

In a viscous fluid it is another empirical fact that the velocity is continuous everywhere, and in particular that the *tangential* component of the velocity of the fluid at the interface is equal to that of the interface - the *no-slip condition*. Hence

$$\mathbf{u} = \mathbf{U} \qquad (18)$$

at the interface (**u** = 0 on a solid boundary at rest).

There are boundary conditions on stress as well as on velocity. In general they can be summarised by the statement that the stress **F** (eq.9) must be continuous across every surface (not the stress tensor, note, just $\underset{\approx}{\sigma}$.**n**), a condition that follows from Newton's third law. At a solid boundary this condition tells you what the force per unit area is and the total stress force on the boundary as a whole is obtained by integrating the stress over the boundary (thus the total force exerted by the fluid on an immersed solid body can be calculated).

When the fluid of interest is water, and the boundary is its interface with the air, the dynamics of the air can often be neglected and the atmosphere can be thought of as just exerting a pressure on the liquid. Then the boundary conditions on the liquid's motion are that its pressure (modified by a small viscous normal stress) is equal to atmospheric pressure and that the viscous shear stress is zero.

CONSEQUENCES: PHYSICAL PHENOMENA

Hydrostatics

We consider a fluid at rest in the gravitational field, with a free upper surface at which the pressure is atmospheric. We choose a coordinate system *x, y, z* such that *z* is measured vertically upwards, so $g_x = g_y = 0$ and $g_z = -g$, and we choose $z = 0$ as the level of the free surface. The density ρ may vary with height, *z*. Thus all components of **u** are zero, and pressure $p = p_{atm}$ at $z = 0$. The Navier-Stokes equations (16) reduce simply to

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0, \quad \frac{\partial p}{\partial z} = -\rho g.$$

Hence

$$p = p_{atm} + g\int_z^0 \rho dz \qquad (19)$$

or, for a fluid of constant density,

$$p = p_{atm} - g\rho z :$$

the pressure increases with depth below the free surface (*z* increasingly negative).

The above results are independent of whether there is a body at rest submerged in the fluid. If there is, one can calculate the total force exerted by the fluid by integrating the pressure, multiplied by the appropriate component of the normal vector **n**, over the body surface. The result is that, whatever the shape of the body, the net force is an upthrust and equal to *g* times the mass of fluid displaced by the body. This is *Archimedes' principle*. If the fluid density is uniform, and the body has uniform density ρ_b, then the net force on the body, gravitational and upthrust, corresponds to a downwards force equal to

$$(\rho_b - \rho)Vg \qquad (20)$$

where *V* is the volume of the body. The quantity $(\rho_b - \rho)$ is called the *reduced density* of the body.

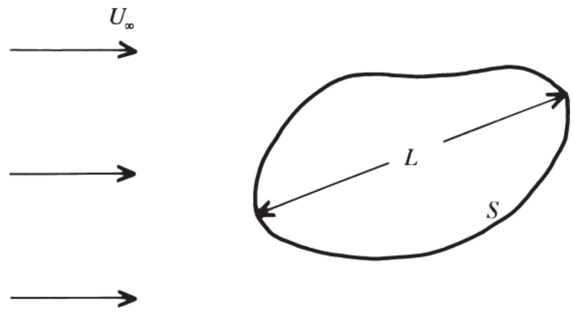

FIG. 6. – Flow of a uniform stream with velocity U_∞ in the *x*-direction past a body with boundary *S* which has a typical length scale *L*.

Note that, for constant density problems in which the pressure does not arise explicitly in the boundary conditions (e.g. at a free surface), the gravity term can be removed from the equations by including it in an *effective pressure*, p_e. Put

$$p_e = p + g\rho z \qquad (21)$$

in equations (16) (with $g_x = g_y = 0$, $g_z = -g$) and see that *g* disappears from the equations, as long as p_e replaces *p*.

Flow past bodies

The flow of a homogeneous incompressible fluid of density ρ and viscosity μ past bodies has always been of interest to fluid dynamicists in general and to oceanographers or ocean engineers in particular. We are concerned both with fixed bodies, past which the flow is driven at a given speed (or, equivalently, bodies impelled by an external force through a fluid otherwise at rest) and with self-propelled bodies such as marine organisms.

Non-dimensionalisation: the Reynolds number

Consider a fixed rigid body, with a typical length scale *L*, in a fluid which far away has constant, uniform velocity U_∞ in the *x*-direction (fig. 6). Whenever we want to consider a particular body, we choose a sphere of radius *a*, diameter $L = 2a$. The governing equations are (8) and (16), and the boundary conditions on the velocity field are

$$u = v = w = 0 \quad \text{on the body surface, S} \qquad (22)$$

$$u \to U_\infty, v \to 0, w \to 0 \quad \text{at infinity.} \qquad (23)$$

Usually the flow will be taken to be steady, ie $\frac{\partial}{\partial t} \equiv 0$, but we shall also wish to think about development of the flow from rest.

For a body of given shape, the details of the flow (i.e. the velocity and pressure at all points in the fluid, the force on the body, etc) will depend on U_∞, L, μ and ρ as well as on the shape of the body. However, we can show that the flow in fact depends only on one dimensionless parameter, the *Reynolds number*

$$Re = \frac{\rho L U_\infty}{\mu}, \tag{24}$$

and not on all four quantities separately, so only one range of experiments (or computations) would be required to investigate the flow, not four. The proof arises when we express the equations in *dimensionless* form by making the following transformations:

$$x' = x/L,\ y' = y/L,\ z' = z/L,\ t' = U_\infty t/L,$$

$$u' = u/U_\infty,\ v' = v/U_\infty,\ w' = w/U_\infty,\ p' = p/\rho U_\infty^2.$$

Then the equations become: (8b):

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} + \frac{\partial w'}{\partial z'} = 0; \tag{25}$$

(16a), with $\frac{Du}{Dt}$ replaced by (4a):

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} + w'\frac{\partial u'}{\partial z'} = -\frac{\partial p'}{\partial x'} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 u'}{\partial z'^2}\right) \tag{26}$$

and there are similar equations starting with $\partial v'/\partial t'$, $\partial w'/\partial t'$. The boundary condition (22) is unchanged, though the boundary S is now non-dimensional, so its shape is important but L no longer appears. Boundary condition (23) becomes

$$u' \to 1, v' \to 0, w' \to 0 \quad \text{at infinity.} \tag{27}$$

Thus Re is the only parameter involving the physical inputs to the problem that still arises.

The drag force on the body (parallel to U_∞) proves to be of the form:

$$D = \tfrac{1}{2}\rho U_\infty^2 A C_D \tag{28}$$

where A (proportional to L^2)is the frontal area of the body ($\pi L^2/4$ for a sphere) and C_D is called the *drag coefficient*. It is a dimensionless number, computed by integrating the dimensionless stress over the surface of the body.

From now on time and space do not permit derivation of the results from the equations. Results will be quoted, and discussed physically where appropriate.

It can be seen from (26) that, in order of magnitude terms, Re represents the ratio of the non-linear inertia terms on the left hand side of the equation to the viscous terms on the right. The flow past a rigid body has a totally different char- acter according as Re is much less than or much greater than 1.

Low Reynolds number flow

When Re <<1, viscous forces dominate the flow and inertia is negligible. Reverting to dimensional form, the Navier-Stokes equations (16d) reduce to the Stokes equations

$$\nabla p_e = \mu \nabla^2 \mathbf{u}, \tag{29}$$

where gravity has been incorporated into p_e using eq. (21). The conservation of mass equation div **u** = 0, is of course unchanged. Several important conclusions can be deduced from this linear set of equations (and boundary conditions).

(i) **Drag** The force on the body is linearly related to the velocity and the viscosity: thus, for example, the drag is given by

$$D = k\mu U_\infty L \tag{30}$$

for some dimensionless constant k (thus the drag coefficient C_D is inversely proportional to Re). In particular, for a sphere of radius a, $k = 3\pi$, so

$$D = 6\pi\mu U_\infty a \tag{31}$$

It is interesting to note that the pressure and the viscous shear stress on the body surface contribute comparable amounts to the drag. The net gravitational force on a sedimenting sphere of density ρ_b, from (20), is $(\rho_b - \rho)\cdot 4/3\pi a^3 g$. This must be balanced by the drag, $6\pi U_s a$, where U_s is the sedimentation speed. Equating the two gives

$$U_s = \frac{2}{9}\frac{(\rho_b - \rho)ga^2}{\mu} \tag{32}$$

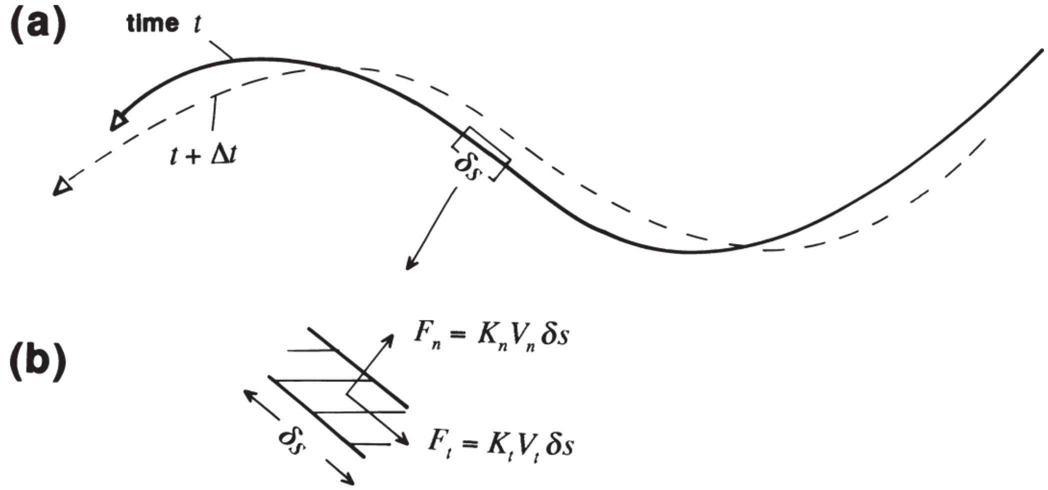

FIG. 7. – (**a**) Sketch of a swimming spermatozoon, showing its position at two successive times and indicating that, while the organism swims to its left, the wave of bending on its flagellum propagates to the right. (**b**) Blow up of a small element δs of the flagellum indicating the force components normal and tangential to it, proportional to the normal and tangential components of relative velocity.

For example, a sphere of radius 10 μm, with density 10% greater than water (ρ=10^{3}kg m^{-3}, $\mu\approx$11kg m^{-1}s^{-1}) will sediment out at only 20 μm^{-1}, whereas if the radius is 100 μm, the sedimentation speed will be 2 mms^{-1}.

(ii) **Quasi-steadiness**. Because the $\partial/\partial t$ term in the equations vanishes at low Reynolds number, it is immaterial whether the relative velocity of the body (or parts of it) and the fluid is steady or not. The flow at any instant is the same as if the boundary motions at that instant had been maintained steadily for a long time - i.e. the flow (and the drag force etc) is *quasi-steady*.

(iii) **The far field**. It can be shown that the far field flow, that is the departure of the velocity field from the uniform stream U_∞, dies off very slowly as the distance r from an origin inside the body becomes large. In fact it dies off as $1/r$, much more slowly, for example, than the inverse square law of Newtonian gravitation or electro-statics. This has an important effect on particle - particle interactions in suspensions. Moreover, this far field flow is proportional to the net force vector $-\mathbf{D}$ exerted *by* the body *on* the fluid, independent of the shape of the body. Thus, in vector form, we can write

$$\mathbf{u} - \mathbf{U}_\infty \approx \frac{1}{r}\left(\mathbf{P} + \frac{(\mathbf{P}.\mathbf{x})\mathbf{x}}{r^2}\right) \qquad (33)$$

where

$$\mathbf{P} = \frac{-\mathbf{D}}{8\pi\mu}. \qquad (34)$$

Measuring the far field is therefore one potential way of estimating the force on the body.

The only exception to the above is the case where the net force on the body (or fluid) is zero, as for a *neutrally buoyant*, self-propelled micro-organism. In that case **P** is zero, the far field dies off like $1/r^2$, and it does depend on the shape of the body and the details of how it is propelling itself.

(iv) **Uniqueness and Reversibility**. If **u** is a solution for the velocity field with a given velocity distribution $\mathbf{u}_s$ on the boundary S, then it is the only possible solution (that seems obvious, but is not true for large Re). It also follows that $-\mathbf{u}$ is the (unique) velocity field if the boundary velocities are reversed, to $-\mathbf{u}_s$. Thus if a boundary moves backwards and forwards reversibly, all elements of the fluid will also move backwards and forwards reversibly, and will not have moved, relative to the body, after a whole number of cycles. Hence a micro-organism *must* have an irreversible beat in order to swim.

(v) **Flagellar propulsion**. Many micro-organisms swim by beating or sending a wave down one or more *flagella*. Fig. 7 sketches a monoflagellate (e.g. a spermatozoon). It sends a, usually helical, wave along the flagellum from the head. This is a non-reversing motion because the wave constantly propagates along. The reason that such a wave can produce a net *thrust*, to overcome the *drag* on the head (and on the tail too) is that about twice as much force is generated by a

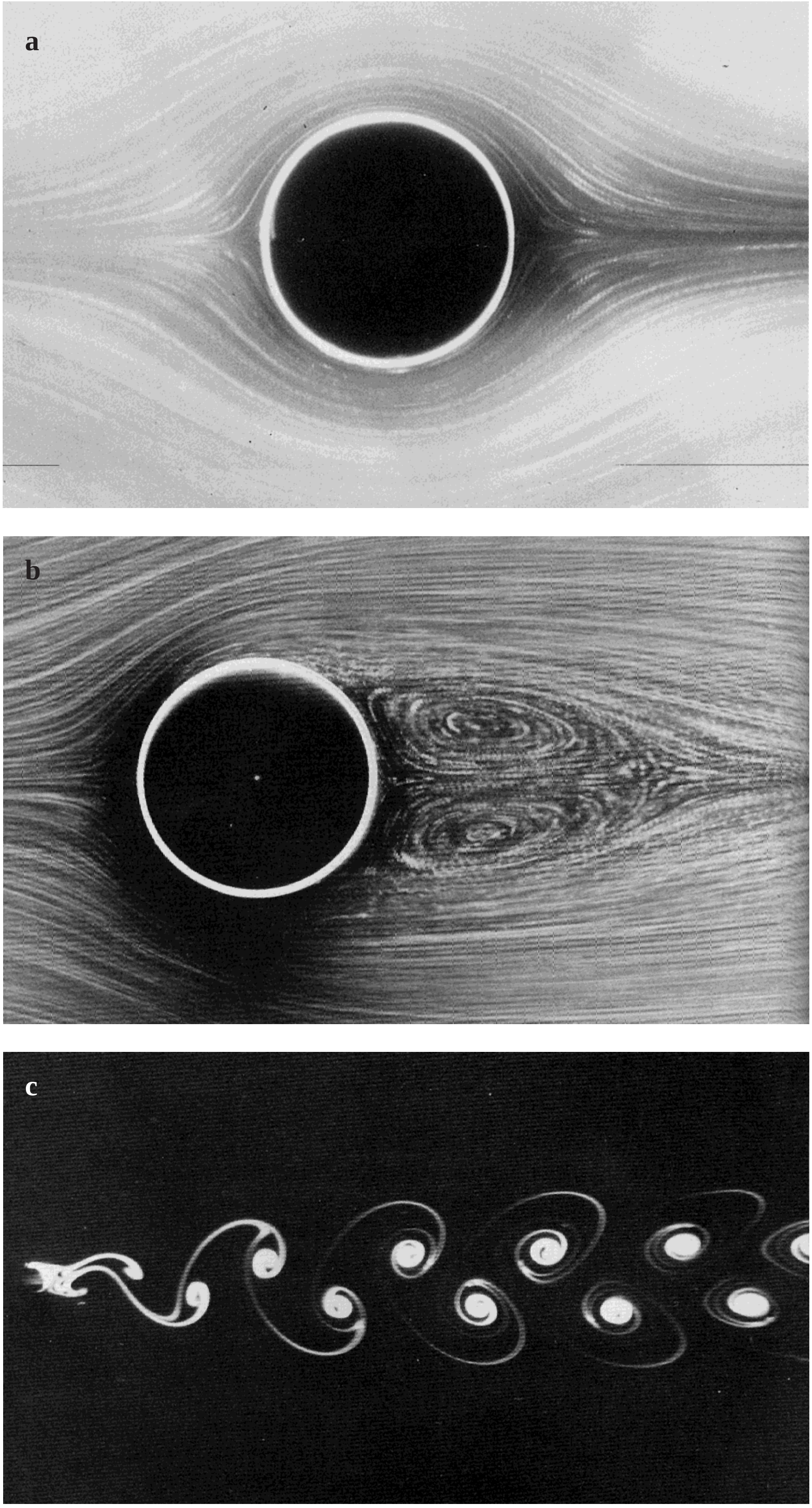

FIG. 8. – Photographs of streamlines (a, b) or streaklines (c) for steady flow past a circular cylinder at different values of the Reynolds number (M.Van Dyke, 1982): (**a**) $Re \ll 1$, (**b**) $Re \approx 26$, (**c**) $Re \approx 105$.

segment of the flagellum moving perpendicular to itself relative to the water as is generated by the same segment moving parallel to itself. This fact forms the basis of *resistive force theory* for flagellar propulsion, which is a simple and reasonably accurate model for the analysis of flagellar locomotion.

Vorticity

The dynamics of fluid flow can often be most deeply understood in terms of the *vorticity*, defined by equation (11) and representing the local rotation of fluid elements. High velocity gradients correspond to high vorticity (see fig. 4). If we take the curl of every term in the Navier-Stokes equation we obtain the following vorticity equation (in vector notation):

$$\frac{\partial \omega}{\partial t} + (\mathbf{u}.\nabla)\omega = (\omega.\nabla)\mathbf{u} + \nu\nabla^2\omega + \frac{1}{\rho^2}\nabla\rho \wedge \nabla p \quad (35)$$

where $\nu = \mu/\rho$ is the *kinematic viscosity* of the fluid (assumed constant). This equation tells us that the vorticity, evaluated at a fluid element locally parallel to ω, changes, as that element moves, as a result of three effects, each repre- sented by one of the terms on the right hand side of (35). The first term can be shown to be associ- ated with rotation and stretching (or compres- sion) of the fluid element, so that the direction of ω remains parallel to the original fluid element, and increases in proportion as the length of that element changes. Such *vortex-line stretching* is a dominant effect in the generation and maintenance of turbulence. It is totally absent in a two-dimensional (2D) flow in which there is no velocity component in one of the coordinate directions (say z) and the variables are independent of z. The second term represents the effect of viscosity, and is *diffusion*-like in that vorticity tends to spread out from elements where it is high to those where it is low. The last term comes about only in non-uniform (e.g. stratified) fluids, and can be important in some oceanographic situations.

It can also be shown that, in a flow started from rest, no vorticity develops anywhere until viscous diffusion has had an effect there. As we shall see, the only *source* of vorticity, in such a flow and in the absence of the last term in (35), occurs at solid boundaries on account of the no-slip condition.

Higher Reynolds number.

It is convenient now to restrict attention to a 2D flow of a homogeneous fluid past a 2D body such as a circular cylinder (fig. 8). In such a 2-D flow, with velocity components $\mathbf{u} = (u,v,0)$, functions of x,y and t, the vorticity is entirely in the third, z, direction, and is given by

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$

There is no vortex-line stretching, and the only effect which can generate vorticity anywhere is viscosity. Let us suppose that the uniform stream at infinity is switched on from rest at the initial instant. Initially there is no vorticity anywhere, and the initial *irrotational* velocity field is easy to calculate. It satisfies all the governing equations and all boundary conditions *except* the no-slip condition at the cylinder surface. The predicted slip velocity therefore generates an infinite velocity gradient $\partial u/\partial y$ and hence a thin sheet, of infinite vorticity at the cylinder surface. Because of viscosity, this immediately starts to diffuse out from the surface. At low values of *Re*, when viscosity is dominant and the convective term $(\mathbf{u}.\nabla)\omega$ in (35) is negligible, the diffusion is rapid, and vorticity spreads out a long way in all directions. An eventual steady state is set up in which the flow is almost totally symmetric front-to-back (fig. 8a); unlike the spherical case, the drag coefficient is not quite inversely proportional to the Reynolds number:

$$C_D = \frac{8\pi}{Re \log(7.4/Re)}.$$

At somewhat higher values of *Re*, the $(\mathbf{u}.\nabla)\omega$ term is not totally negligible, and once vorticity has reached any particular fluid element it tends to be carried along by it as well as diffusing on to other elements. Hence a front-to-back asymmetry develops. For *Re* greater than about 5 the flow actually separates from the wall of the cylinder, forming two slowly recirculating flow regions (eddies) at the rear. At still higher *Re*, it is observed that the eddies tend to break away alternately from the two sides of the cylinder, usually at a well-defined frequency equal to about 0.42 U_∞/a for $Re \geq 600$, and steady flow is no longer possible. At higher *Re* the wake becomes *turbulent* (i.e. random and three-dimensional) and at $Re \approx 2 \times 10^5$ the flow on the cylinder surface becomes turbulent.

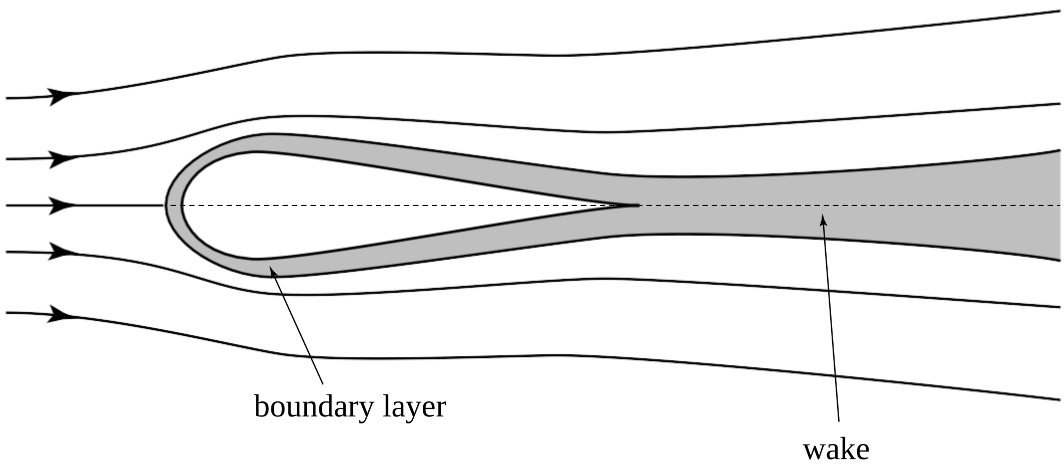

FIG. 9. – Sketch of boundary layer and wake for steady flow at high Reynolds number past a symmetric streamlined body.

Steady flows at relatively high Reynolds number do seem to be possible past streamlined bodies such as a wing (or a fish dragged through the fluid), see fig. 9. Diffusion causes vorticity to occupy a (boundary) layer of thickness $(\nu t)^{1/2}$ after time t. However, even a fluid element near the leading edge at first will have been swept off downstream past the trailing edge after a time $t = L/U_\infty$, where L is the length of the wing chord. Hence the greatest thickness that the boundary layer on the body can have is

$$\delta_s = \left(\nu L / U_\infty\right)^{1/2}, \qquad (36)$$

and it is easy to see that a steady state can develop everywhere on the body, with a boundary layer of thickness up to δ_s, and a thin wake region, also containing vorticity, downstream. Note that the boundary layer of vorticity remains thin compared with the chord length if $\delta_s << L$, i.e. $Re >>1$. In that case (and only then) neglecting viscosity altogether, and forgetting about the boundary layer, is accurate enough, except in calculating the drag.

Drag on a symmetric body at large Reynolds number. In order to estimate the force on a body it is necessary to work out the distribution of pressure

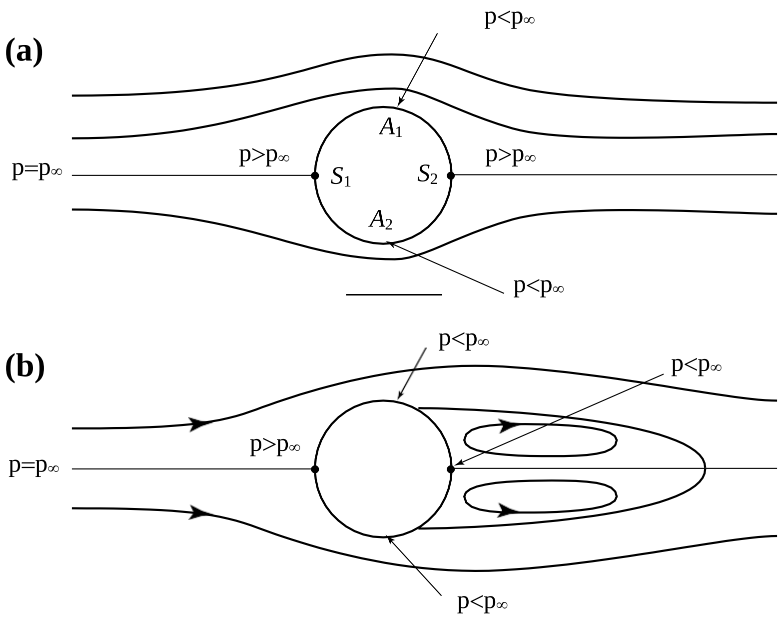

FIG. 10. – Sketch of streamlines and pressures for flow past a circular cylinder. (**a**) Idealised flow of a fluid with no viscosity; (**b**) separated flow at fairly high Reynolds number in a viscous fluid.

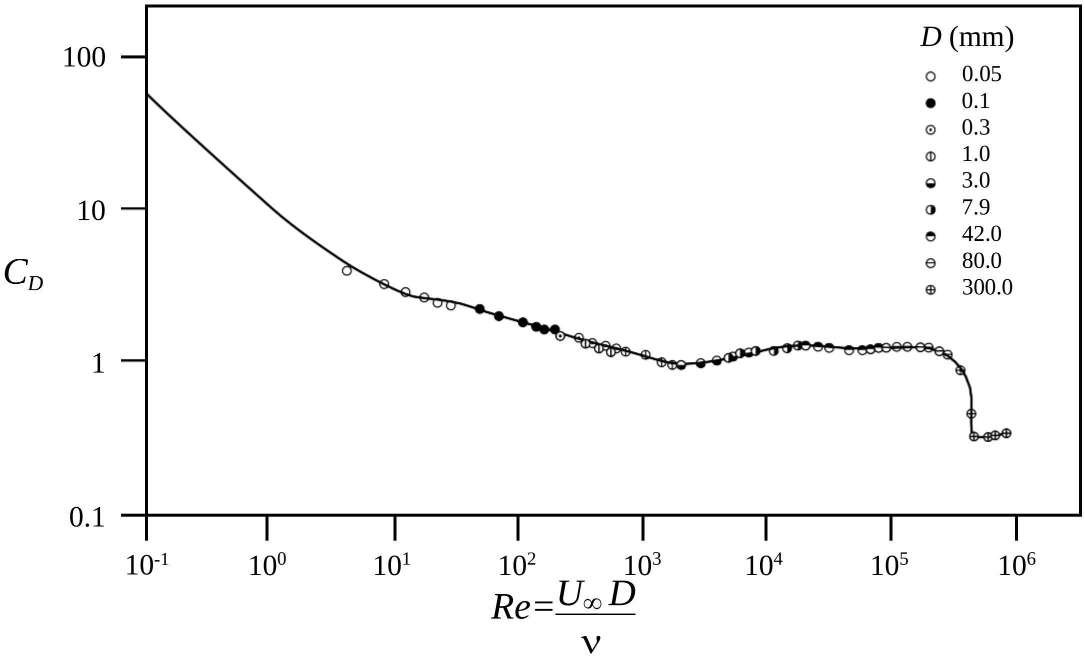

FIG. 11. – Log-log plot of drag coefficient versus Reynolds number for steady flow past a circular cylinder. [The sharp reduction in C_D at $Re \approx 2 \times 10^5$ is associated with the transition to turbulence in the boundary layer]. Redrawn from Schlichting (1968).

round the body. In a steady flow of constant density fluid in which viscosity is unimportant (e.g. outside the boundary layer and wake of a body) equation (16d) can be integrated to give the result that the quantity

$$p + pgz + \frac{1}{2}\rho|\mathbf{u}|^2 = \text{constant} \qquad (37a)$$

along streamlines of the flow. Here z is measured vertically upwards and $|\mathbf{u}|$ is the total fluid speed. This result is equivalent to the Newtonian principle of conservation of energy; equation (37a) is called *Bernoulli's equation*. If we forget about the gravitational contribution, replacing $p + pgz$ by the effective pressure p_e (eq. 21), equation (37a) becomes

$$p_e = \text{constant} - \frac{1}{2}\rho|\mathbf{u}|^2; \qquad (37b)$$

henceforth we just write p for p_e. If the fluid speeds up, the pressure falls, and vice versa, which is intuitively obvious since a favourable pressure gradient is clearly required to give fluid elements positive acceleration.

In the case of flow past a symmetric body, (fig. 10a), all streamlines start from a region of uniform pressure (p_∞ say) and uniform velocity (U_∞), so the constant in (37b) is the same for all streamlines, $p_\infty + 1/2U^2_\infty$. If viscosity were really negligible, then the flow round a circular cylinder would be symmetric (fig. 10a). At the front *stagnation point* S_1, the point of zero velocity where the streamline dividing flow above from flow below impinges, the pressure is high ($p = p_\infty + 1/2\rho U^2_\infty$), and this high pressure is balanced by an equally high pressure at the rear stagnation point S_2. The pressure at the sides (A_1, A_2) is low ($p = p_\infty - 3/2U^2_\infty$). The net effect is that the hydrodynamic force on the cylinder is zero.

In a viscous fluid, as stated above, there is a thin boundary layer on the front half, in which the velocity falls from a large value to zero, so the pressure distribution is similar to that described above; however the flow separates on the rear half and things are very different. The reason for the separation is that the adverse pressure gradient (the pressure rise), from A_1 to S_2 say, causes the low velocity in the boundary layer to tend to reverse its direction, and it is observed that separation occurs as soon as flow reversal takes place. In the separated flow region (fig. 10b) the fluid velocity is low and the pressure remains close to its value at the sides. Thus there is a front-to-back pressure difference proportional to ρU^2_∞, and the drag coefficient C_D (eq. 28) is approximately constant, independent of Re as long as Re is large (see fig. 11). The direct contribution of tangential viscous stresses to the drag is negligibly small, although it is the presence of viscosity which causes the flow separation in the first place.

Lift. For a symmetric streamlined body (fig. 9) flow separation occurs only very near the trailing edge, and direct viscous drag is more important. However, if such a streamlined body (or wing) is tilted so that the oncoming flow makes an angle of incidence with its centre plane, viscosity again has an important effect. In general, a non-viscous flow past a wing at incidence would turn sharply round the trailing edge, where the velocity would be extremely high and the pressure extremely low (fig. 12a). As the flow starts up from rest, viscosity causes separation at the corner, a concentrated vortex is shed and left behind, and thereafter the flow is forced to come tangentially off the trailing edge: the *Kutta-condition* (fig. 12b). In order to achieve this tangential flow, the velocity on top of the wing must increase and the velocity below must decrease. It follows from Bernoulli's equation that the pressure above the wing must fall, and that below rise, so a transverse force is generated. This is call *lift* and keeps aircraft and birds in the air against gravity. The magnitude of the lift is also represented by a *lift coefficient* C_L:

$$L = \frac{1}{2} U_\infty^2 S C_L , \tag{38}$$

where S is the horizontal area of the wing. Like C_D, C_L is approximately independent of Re for large Re.

Added mass. We have seen that the force on a body in an inviscid fluid is zero when the flow is steady. When the flow is unsteady, however, the force is non-zero, because accelerating the body rel- ative to the fluid requires that the fluid also has to be accelerated. Thus the body exerts a force on the fluid and so, by Newton's third law, the fluid exerts an equal and opposite force on the body. In all cases, this force is equal in magnitude to the acceleration of the body relative to the fluid multiplied by the mass of fluid displaced by the body (ρV in the notation of eq. 20) multiplied by a constant, say β:

$$F = \beta \rho V \, dU/dt. \tag{39}$$

For a sphere, $\beta = 0.5$; for a circular cylinder, $\beta = 1$. The quantity $\beta \rho V$ is call the *added mass* of the body in question (recall that ρ is the *fluid* density). The corresponding force, given by (39), is called the *acceleration reaction*, or the *reactive force*.

Fish swimming. We have seen that flagellates such as spermatozoa swim by sending bending waves down their tails, and thrust is generated through the viscous, resistive force. Inertia is negligible because the Reynolds number is small. For most fish, the Reynolds number is large, but nevertheless many fish also swim by sending a bending wave down their bodies and tails. In this case, however, thrust is generated primarily by the reactive force associated with the sideways acceleration of the elements of fluid as they pass down the animal (relative to a frame of reference fixed in the fish's nose). Lighthill has developed a simple, reactive-force model for fish swimming.

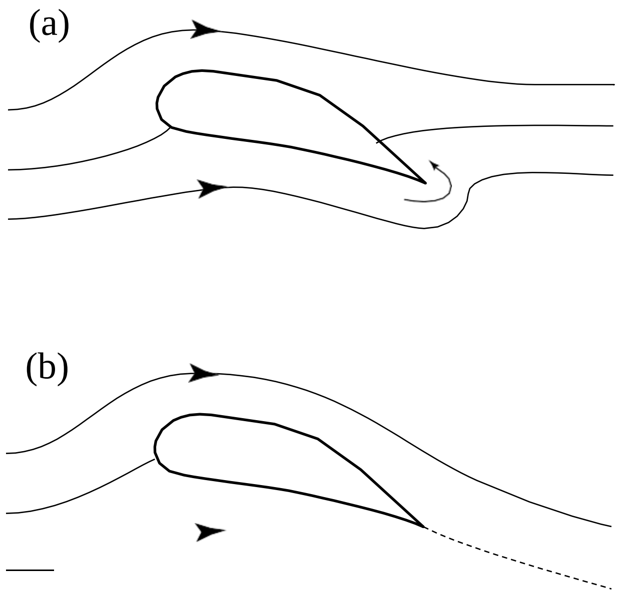

FIG. 12. – Flow past a streamlined body at incidence. (**a**) Idealised flow of a fluid with no viscosity - large velocity and pressure gradient round the trailing edge. (**b**) In a viscous fluid the flow must come smoothly off the trailing edge, which explains the generation of lift (see text).

Flow in the open ocean

Water waves

The most obvious dynamical feature of the ocean, to even a casual observer, is the presence of surface *waves*, of a variety of lengths and heights. Waves are mainly generated as a result of stresses exerted by the wind, although they can also arise through the impact or relative motion of foreign bodies such as rain drops or ships. Once generated, however, waves can propagate over large distances and persist for long times, unaffected by the atmosphere or solid bodies.

In a periodic wave motion, all fluid elements affected by it experience oscillations. Like all oscillations, such as that of a simple pendulum, these oscillations come about as an interaction between a *restoring force*, tending to restore a particle to a nearby equilibrium position, and *inertia*, which causes the particle to overshoot each time it reaches its equilibrium position (in real systems there is also some viscous *damping*, which causes the amplitude of the oscillations to die out after a long time, if there is no further stimulation; we ignore damping here). In the case of a simple pendulum (a mass suspended by a light string) the equilibrium state is one in which the string is vertical and the mass at rest, the restoring force is gravity and the inertia is the momentum of the mass itself. In the case of water waves, the equilibrium state has the free surface horizontal, the restoring force is again gravity (except for small wavelengths, when surface tension is also important) and the inertia is the momentum of the fluid. Viscosity is negligible because there are no solid boundaries generating vorticity.

In an oscillation of small amplitude, every particle exhibits *simple harmonic motion*: its vertical displacement, say Y, from equilibrium, varies with time according to the differential equation

$$\frac{d^2Y}{dt^2} + \omega^2 Y = 0. \qquad (40)$$

The general solution for Y is a sinusoidal oscillation of the form

$$Y = A\cos(\omega t - \phi)$$

where A and ϕ are constants (determined by initial conditions), the amplitude and phase respectively, and ω is the *angular frequency* of the oscillation (the frequency in Hertz is $\omega/2\pi$). In the case of a simple pendulum, $\omega = (g/l)^{1/2}$ where l is the length of the string. In the case of simple water waves of wave length $\lambda = 2\pi/k$ (k is the *wave number*), in an ocean whose depth is much greater than λ, we have

$$\omega = (gk)^{1/2} \qquad (41)$$

as long as surface tension is negligible.

Suppose a parallel-crested (one-dimensional) train of such waves is propagating in the x-direction. Then the displacement of the free surface will be given by

$$\eta = A\cos(\omega t - kx - \phi), \qquad (42)$$

again for constant A and ϕ. The speed of propagation of the wave crests, or *phase velocity*, is

$$c = \frac{\omega}{k} = \left(\frac{g}{k}\right)^{1/2}. \qquad (43)$$

Thus long waves (small k) travel more rapidly than short waves (large k). This explains why, when the waves are generated by a localised disturbance, such as a storm at sea, or a stone dropped in a pond, the longer waves (swell) arrive at the shore first. In this case, the wave front travels at a different speed, called the group velocity, c_g:

$$c_g = \frac{d\omega}{dk} = \frac{1}{2}\left(\frac{g}{k}\right)^{1/2}, \qquad (44)$$

so that wave crests, travelling faster, appear to arise at the back of the packet of waves, and to disappear at the front.

When a water wave propagates, with its free surface given by (42), fluid elements at and below the surface move in circular paths, and the amplitude of their motion falls off exponentially with depth below the surface: the amplitude is proportional to Ae^{kz} when the undisturbed surface is at $z = 0$. Thus the amplitude is negligibly small at a depth of only half a wavelength ($kz = -\pi$). This explains why the theory of waves in very deep water works well in relatively shallow water, too, with depth h greater than half a wavelength. When the waves are very long, or the water very shallow, equation (41) is replaced by

$$\omega = (gk \tanh kh)^{1/2} \qquad (45)$$

Small amplitude wave theory is very useful, because the equations are linear and a general motion can be made up from the addition of many sinusoidal components such as (42) (a Fourier series or transform). At larger amplitudes, nonlinear effects become important and the theory becomes less general, although many interesting and important phenomena arise, such as wave breaking.

Internal waves

Although the water in the ocean is effectively incompressible, it does not have uniform density because it is *stratified* on account of the variation with depth of the pressure and, to a lesser extent, the temperature and the salinity. The temperature/density distribution is marked usually by one or more thermoclines, in which the density gradient is steeper than elsewhere. Whether the density gradient is uniform or locally sharp, less dense fluid sits, in equilibrium, above denser fluid. A disturbance to this state causes some heavy fluid elements to rise above their original level, and some light ones to fall below. As in the case of surface waves, gravity then provides a restoring force and *internal gravity waves* can propagate. As for surface waves, a relation can be calculated between the frequency and the wave number of such waves. For example, if there is a sharp interface between two deep regions of fluid with densities ρ_1 (above) and ρ_2, then equation (41) is replaced by

$$\omega^2 = gk\left[(\rho_2 - \rho_1)/(\rho_2 + \rho_1)\right]. \qquad (46)$$

This can be seen to give much lower frequencies than (41) if $(\rho_2 - \rho_1)$ is not large: if $\rho_2 - \rho_1 = 0.1\,\rho_2$, then the frequency given by (46) is 4.4 times smaller than that given by (41) (with $\rho_2 = \rho$). The propagation speed is correspondingly smaller, too.

When the density gradient is uniform, with

$$\frac{g\,d\rho}{\rho\,dz} = -N^2, \qquad (47)$$

where N is a constant with the dimensions of a frequency (the *Brunt -Väisälä* frequency), the situation is a bit more complicated, because internal waves do not have to propagate horizontally. Indeed, a wave whose crests propagate at an angle θ to the horizontal, so that the displacement of a fluid element is given by

$$y = A\cos\left[\left(\omega t - k(x\cos\theta + z\sin\theta)\right)\right],$$

has a frequency ω given by

$$\omega = N\cos\theta\,. \qquad (48)$$

However, the group velocity (velocity of a wave front, or of energy propagation) is perpendicular to the phase velocity, and in this case is given by the vector

$$\mathbf{c}_g = \frac{N}{k}\sin\theta(\sin\theta, 0, -\cos\theta). \qquad (49)$$

Rotating fluids: geostrophic flows

Gravity waves are (mostly) small-scale phenomena for which the rotation of the earth is unimportant. That is not the case with ocean currents and the large-scale circulation of the oceans. To analyse such motions, it is necessary to recognise that the natural frame of reference is fixed in the rotating earth, and the governing equations of motion have to be changed accordingly. If viscosity is neglected, the equation of motion of a fluid in a frame of reference rotating with constant angular velocity Ω becomes (in place of (16d)):

$$\rho\frac{D\boldsymbol{u}}{Dt} + \rho\Omega\wedge\boldsymbol{u} = \rho\mathbf{g} - \nabla p. \qquad (50)$$

Here **g** has been modified to include the small "centrifugal force'' term, and we could also incorporate it into the pressure using (21). The additional term $\rho\Omega\wedge\boldsymbol{u}$ is called the *Coriolis force*.

Time does not permit a thorough investigation of the dynamics of rotating fluids. We consider only a flow in which the Coriolis force is much larger than the other inertia terms and therefore must by itself balance the gradient in (effective) pressure: a *geostrophic flow*. For such a flow, (50) reduces to

$$\rho\Omega\wedge\boldsymbol{u} = -\nabla p. \qquad (51)$$

Suppose the flow is horizontal: $\mathbf{u} = (u, v, 0)$, with z vertically upwards again. Then the horizontal components of the pressure gradient are given by

$$\frac{\partial p}{\partial x} = -\Omega_\upsilon v, \quad \frac{\partial p}{\partial y} = +\Omega_\upsilon u \qquad (52)$$

where Ω_υ is the vertical component of the earth's angular velocity (total angular velocity multiplied by the sine of the latitude). The pressure gradient is perpendicular to the velocity, or vice versa, indicating that if there is a horizontal pressure gradient, the corresponding geostrophic flow will be perpendicular to it. This explains why the wind goes anticlockwise round atmospheric depressions in the northern hemisphere (clockwise in the southern hemisphere). Similar flows occur in the oceans, although the barriers formed by the continents are impermeable, unlike in the atmosphere.

The condition for a steady flow to be geostrophic is that the inertia term $(\boldsymbol{u}.\nabla)\boldsymbol{u}$ should be small compared with the Coriolis term. Thus if U is a typical velocity magnitude, and L a length scale for the flow, the geostrophic approximation will be a good one if

$$\frac{U^2}{L} << \Omega_v U,$$

i.e. the *Rossby number* should be small:

$$\frac{U^2}{\Omega_v L} << 1. \qquad (53)$$

If the Rossby number is large, the earth's rotation can be neglected. Note that the Rossby number is always large at the equator, where $\Omega_v = 0$.

Hydrodynamic instability

A smooth, laminar flow becomes turbulent as a result of hydrodynamic instability. Small, random perturbations are inevitably present in any real system; if they die away again, the flow is stable, but if they grow large, the original flow becomes unrecognisable and is unstable. Usually, steady flows which are slow or weak enough are stable, but they become unstable above some critical speed or strength.

The way to investigate instabilities mathematically is to assume that the disturbances to the steady state are very small and to linearise the equations accordingly. Thus, if the steady state velocity, pressure and density are given by $\mathbf{u}_0(\mathbf{x})$, $p_0(\mathbf{x})$ and $\rho_0(\mathbf{x})$ (all functions of position, in general) it is postulated that, with the perturbation, we have

$$\mathbf{u} = \mathbf{u}_0(\mathbf{x}) + \mathbf{u}'(\mathbf{x},\ t),$$
$$p = p_0(\mathbf{x}) + p'(\mathbf{x},\ t),$$
$$\rho = \rho_0(\mathbf{x}) + \rho'(\mathbf{x},\ t)$$

where $\mathbf{u}'$, p' and ρ' are small. Then these are substituted into the governing equations, and terms involving squares or products of small quantities are neglected, so the equations are linearised. For example, equation (7) which, with (3b), is

$$\frac{\partial\rho}{\partial t} + \mathbf{u}.\nabla\rho = 0,$$

becomes

$$\frac{\partial\rho'}{\partial t} + \mathbf{u}_0.\nabla\rho' + \mathbf{u}'.\nabla\rho_0 = 0 \qquad (54)$$

and the nonlinear term $\mathbf{u}'\nabla\rho'$ is neglected. After linearisation, it is usually possible to think of the disturbance as made up of many *modes* in which the variables depend sinusoidally on one or more space coordinates and exponentially on time, e.g.

$$\rho' = f(z)\exp\left\{i(kx + ly) + \sigma t\right\} \qquad (55)$$

(cf 42), where $i = \sqrt{-1}$ and we are using complex number notation. Such terms are substituted into the equations, and it turns out that a solution of the supposed form exists only if σ takes a particular value. If that value has negative (or zero) real part, the disturbance dies away (or oscillates at constant amplitude); if it has positive real part it grows exponentially, indicating instability. If *any* disturbance of the form (55) (i.e. for any values of k and l) grows, then the flow is unstable, because in general all disturbances are present, infinitesimally, at first.

Consider, for example, the case of two fluids of different densities, one on top of the other. We have seen that the frequency of a disturbance of wavenumber k is given by equation (46) if ρ_2(the density of the lower fluid) is greater than ρ_1.

^{2}However, if $\rho_1 > \rho_2$, ω as given by (46) is negative. But if we replace ω by $i\sigma$, σ^2 is positive, σ is real, and the oscillation $\cos\omega t$ can be written as $1/2(e^{\sigma t} + e^{-\sigma t})$.Thus exponential growth is predicted. Hence the interface between a dense fluid and a less dense fluid below it is unstable.

A similar analysis can be performed for a continuous density distribution, denser on top, caused by a temperature gradient, say, in a fluid heated from below. In this case the diffusion of heat (and hence density) must be allowed for, as well as conservation of fluid mass and momentum. For example, a horizontal layer of fluid, contained between two rigid horizontal planes, distance h apart and maintained at temperatures T_0 (top) and $T_0 + \Delta T$ (bottom) is unstable if the temperature difference ΔT is large enough. More precisely, instability occurs if a dimensionless parameter called the *Rayleigh number Ra* exceeds the critical value of 1708, where

$$Ra = \frac{g\alpha\Delta T h^3}{\nu\kappa} \qquad (56)$$

Here α, ν and κ are fluid properties, the coefficient of expansion, the kinematic viscosity (μ/ρ) and the thermal diffusivity respectively. When instability occurs, for values of Ra not much greater than 1708, the resulting motion is a regular array of usually hexagonal cells (fig. 13), with fluid flow up in the centre of the cells and down at the edges. Such a motion is an example of *thermal convection*, called *Rayleigh-Benard* convection. When Ra is much higher than 1708, the cells themselves become unstable, the convection becomes very complicated and eventually turbulent.

Rayleigh-Benard convection is an example in which instability of the original steady state leads to another, regular, steady motion which itself goes

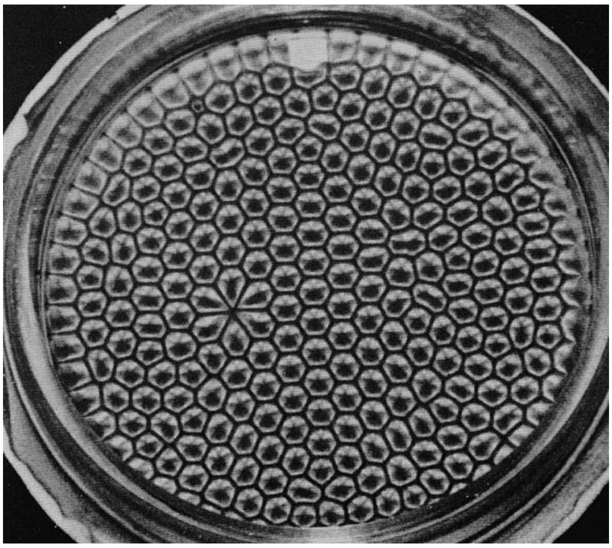

FIG. 13. – Photograph of convection pattern for Rayleigh-Benard convection in a layer of fluid heated from below. (Koschmieder, 1974).

unstable as Ra is increased, and turbulence results only after a whole sequence of such instabilities, or bifurcations. Other systems do not seem to have intermediate stable steady states, but there is a rapid transition from laminar to turbulent flow when critical conditions are passed. Perhaps the most familiar and important of such flows are unidirectional (or approximately so) *shear flows*, such as that depicted in fig. 4. Examples are flow in a straight pipe and flow in the boundary layer on a rigid body or in the shear layer at the edge of the recirculation behind it. Flow in a circular pipe of diameter D normally becomes turbulent when the Reynolds number

$$Re = \frac{D\bar{u}}{\nu},$$

where $\bar{u}$ is the cross-sectionally averaged velocity, exceeds a critical value of just over 2000. Flow in a boundary layer on a thin flat plate (an approximation to a streamlined body) becomes unstable when the Reynolds number based on the free stream velocity and the boundary layer thickness δ_s (eq. 36) exceeds about 244. Flow in a shear layer is more unstable still, associated with the fact that the velocity profile contains an inflection point.

When numbers are put in to formulae such as those quoted above, it becomes clear that oceanic flows are necessarily turbulent. Hence the existence of this course.

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CONCEPT OF CONTROL VOLUME

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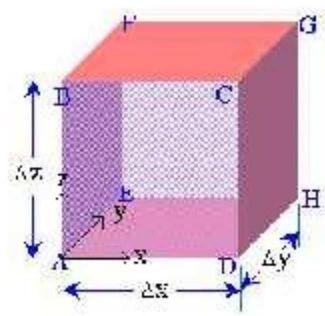
CONTINUITY EQUATION Concepts

The continuity equation is governed from the principle of conservation of mass.It states that the mass of fluid flowing through the pipe at the cross-section remains constants,if there is no fluid is added or removed from the pipe.

Let us make the mass balance for a fluid element as shown below: (an open-faced cube)



Let us denote the sides by with the following corresponding numbers:

x -direction		y -direction		z -direction	
ABFE	1	ABCD	3	AEHD	5
DCGH	2	EFGH	4	BFGC	6

$$\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = -\frac{\partial \rho}{\partial t}$$

This is the continuity equation for every point in a fluid flow whether steady or unsteady , compressible or incompressible.

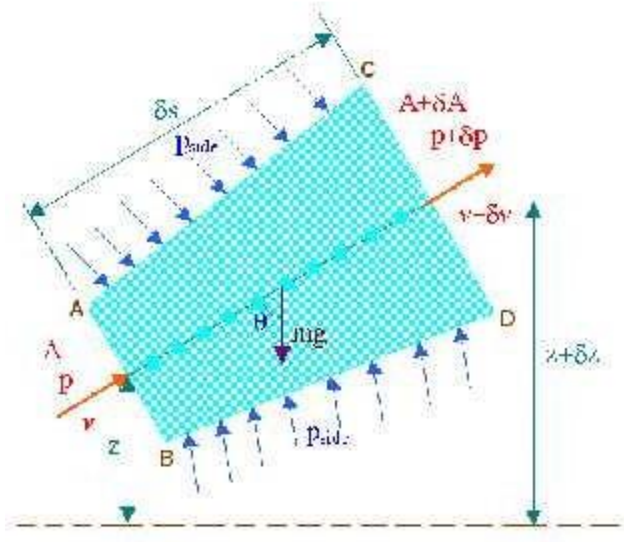
For steady, incompressible flow, the density ρ is constant and the equation simplifies to

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

For two dimensional incompressible flow this will simplify still further to

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

1.1.1 ULER'S EQUATION OF MOTION



$$\frac{1}{\rho}\frac{dp}{ds}+\frac{vdv}{ds}+g\frac{dz}{ds}=0$$

$$\frac{dp}{\rho}+vdv+gdz=0$$

This is known as Euler's equation, giving, in differential form the relationship between p, *v*, ρ and elevation z, along a streamline for steady flow.

1.1.2 BERNOULLI EQUATION

Concepts

Bernoulli's Equation relates velocity, pressure and elevation changes of a fluid in motion. It may be stated as follows " In an ideal incompressible fluid when the flow is steady and continuous the sum of pressure energy, kinetic energy and potential energy is constant along streamline"

$$\frac{p}{\rho g}+\frac{v^2}{2g}+z=\text{constant}$$ --> 1

This is the basic from of *Bernoulli equation* for steady incompressible inviscid flows. It may be written for any two points 1 and 2 on the same streamline as

$$\frac{p_1}{\rho_1 g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho_2 g}+\frac{v_2^2}{2g}+z_2$$ --> 2

The constant of Bernoulli equation, can be named as *total head (h_o)* has different values on different streamlines.

$$h_o = \frac{p}{\rho g} + \frac{v^2}{2g} + z \quad \text{--> 3}$$

The total head may be regarded as the sum of the *piezometric head* $h^* = p/\rho g + z$ and the *kinetic head* $v^2/2g$.

Bernoullie equation is arrived from the following assumptions:

1. Steady flow - common assumption applicable to many flows.
2. Incompressible flow - acceptable if the flow Mach number is less than 0.3.
3. Frictionless flow - very restrictive; solid walls introduce friction effects.
4. Valid for flow along a single streamline; i.e., different streamlines may have different h_o.
5. No shaft work - no pump or turbines on the streamline.
6. No transfer of heat - either added or removed.

Range of validity of the Bernoulli Equation:

Bernoulli equation is valid along any streamline in any steady, inviscid, incompressible flow. There are no restrictions on the shape of the streamline or on the geometry of the overall flow. The equation is valid for flow in one, two or three dimensions.

Modifications on Bernoulli equation:

Bernoulli equation can be corrected and used in the following form for real cases.

$$\frac{p_1}{\rho_1 g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho_2 g} + \frac{v_2^2}{2g} + z_2 + h + w - q$$

APPLICATIONS

1.Venturimeter.

2.Orificemeter

3.Pitot Tube

1.2 MOMENTUM EQUATION

Net force acting on fluid in the direction of x=Rate of change of momentum in x direction

=Mass per sec× Change in

velocity $p1A1-p2A2\times\cos\theta-$

$Fx=\rho Q(v2\cos\theta-v1)$

$$Fx=\rho Q(v1-v2\cos\theta)-p2A2\cos\theta+p1A1$$

Similarlt,the momentum in y-direction is

$-p2A2\sin\theta+Fy=\rho Q(v2\sin\theta-0)$

$$Fy=\rho Qv2\sin\theta+p2A2\ \sin\theta$$

Resultant force acting on the bend,

$Fr=\sqrt{Fx^2+Fy^2}$

GLOSSARY

Quantity	***Unit***
Mass in Kilogram	Kg
Length in Meter	M
Time in Second	s or as sec
Temperature in Kelvin	K
Mole	gmol or simply as mol

Derived quantities:

Quantity	***Unit***
Force in Newton (1 N = 1 kg.m/s^2)	N
Pressure in Pascal (1 Pa = 1 N/m^2)	N/m^2
Work, energy in Joule (1 J = 1 N.m)	J
Power in Watt (1 W = 1 J/s)	W

REVIEW QUESTIONS

PART A

1. Define compressibility of a fluid.
2. What is viscosity? What is the cause of it in liquids and gases.
3. What is the effect of temperature on viscosity of water and that of air?
4. Explain about capillarity.
5. Distinguish between fluid and solid.
6. Define (a) Dynamic viscosity and (b) Kinematic viscosity.
7. Define (a) Surface tension (b) Capillarity
8. What is a real fluid? Give examples.
9. Define cavitation.
10. Define Viscosity
11. Define the following fluid properties:
12. Density, weight density, specific volume and specific gravity of a fluid.

PART B

1. (a) What are the different types fluids? Explain each type. **(**b) Discuss the thermodynamic properties of fluids

2. (a) One litter of crude oil weighs 9.6 N. Calculate its Specific weight, density and specific weight.
(b) The Velocity Distribution for flow over a flat plate is given by u=(2/3)y-y2, Where u is the point velocity in meter per second at a distance y meter above the plate. Determine the shear stress at y=0 and y=15 cm. Assume dynamic viscosity as 8.63 poises

3. **(**a) A plate, 0.025 mm distant from a fixed plate, moves at 50 cm/s and requires a force of 1.471 N/ m2 to maintain this speed. Determine the fluid viscosity between plates in the poise.
(b) Determine the intensity of shear of an oil having viscosity =1.2 poise and is used for lubrication in the clearance between a 10 cm diameter shaft and its journal bearing. The clearance is 1.0 mm and Shaft rotates at 200 r.p.m

4. (a) Two plates are placed at a distance of 0.15mm apart. The lower plate is fixed while the upper plate having surface area 1.0 m2 is pulled at 0.3 nm/s. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity 1.5 poise.
(b) An oil film of thickness 1.5 mm is used for lubrication between a square plate of size 0.9m *0.9m and an inclined plane having an angle of inclination 20^0 . . The weight of square plate is 392.4 N and its slides down the plane with a uniform velocity of 0.2 m/s. find the dynamic viscosity of the oil.

5. (a) Assuming the bulk modulus of elasticity of water is 2.07 x10 6 kN/m2 at standard atmospheric condition determine the increase of pressure necessary to produce one percent reduction in volume at the same temperature
(b) Calculate the capillary rise in glass tube pf 3mm diameter when immersed in mercury, take the surface tension and angle of contact of mercury as 0.52 N/m and 1300 respectively. Also determine the minimum size of the glass tube, if it is immersed in water, given that the surface tension of water is 0.0725 N/m and Capillary rise in tube is not exceed 0.5mm

6. **(**a) Calculate the pressure exerted by 5kg of nitrogen gas at a temperature of 10^0C. Assume ideal gas law is applicable.
(b) Calculate the capillary effect in glass tube 5mm diameter, when immersed in (1) water and (2) mercury. The surface tension of water and mercury in contact with air are 0.0725 N/m and 0.51 N/m respectively. The angle of contact of mercury of mercury is 130.

7. **(**a) Explain all three Simple manometers with neat sketch.
(b) Explain Differential manometer With Neat sketch.

8. A U-tube differential manometer is connected two pressure pipes A and B.Pipe A contains Carbon tetrachloride having a specific gravity 1.594 under a pressure of 11.772 N/ Cm^2 . The pipe A lies 2.5 m above pipe B. Find the difference of pressure measured by mercury as a fluid filling U-tube

UNIT -II FLOW THROUGH CIRCULAR CONDUITS

PRE REQUEST DISCUSSION

Unit II has an in dept dealing of laminar flow through pipes, boundary layer concept, hydraulic and energy gradient, friction factor, minor losses, and flow through pipes in series and parallel.

Boundary layer is the region near a solid where the fluid motion is affected by the solid boundary. In the bulk of the fluid the flow is usually governed by the theory of ideal fluids. By contrast, viscosity is important in the boundary layer. The division of the problem of flow past an solid object into these two parts, as suggested by Prandtl in 1904 has proved to be of fundamental importance in fluid mechanics.

This concept of hydraulic gradient line and total energy line is very useful in the study of flow This concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes. f fluids through pipes.

2.1 HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

1.Hydraulic Gradient Line

It is defined as the line which gives the sum of pressure head (p/w) and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L (Hydraulic Gradient Line).

2.Total Energy Line

It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L (Total Energy Line).

2.2 BOUNDARY LAYER

Concepts

The variation of velocity takes place in a narrow region in the vicinity of solid boundary. The fluid layer in the vicinity of the solid boundary where the effects of fluid friction i.e., variation of velocity are predominant is known as the *boundary layer.*

2.2.1 FLOW OF VISCOUS FLUID THROUGH CIRCULAR PIPE

For the flow of viscous fluid through circular pipe, the velocity distribution across a section, the ratio of maximum velocity to average velocity, the shear stress distribution and drop of pressure fora given length is to be determined. The flow through circular pipe will be viscous or laminar, if the Reynold's number is less than 2000.

2.2.2 DEVELOPMENT OF LAMINAR AND TURBULENT FLOWS IN CIRCULAR PIPES

1. Laminar Boundary Layer

At the initial stage i.e, near the surface of the leading edge of the plate, the thickness of boundary layer is the small and the flow in the boundary layer is laminar though the main stream flows turbulent. So, the layer of the fluid is said to be *laminar boundary layer.*

2. Turbulent Boundary Layer

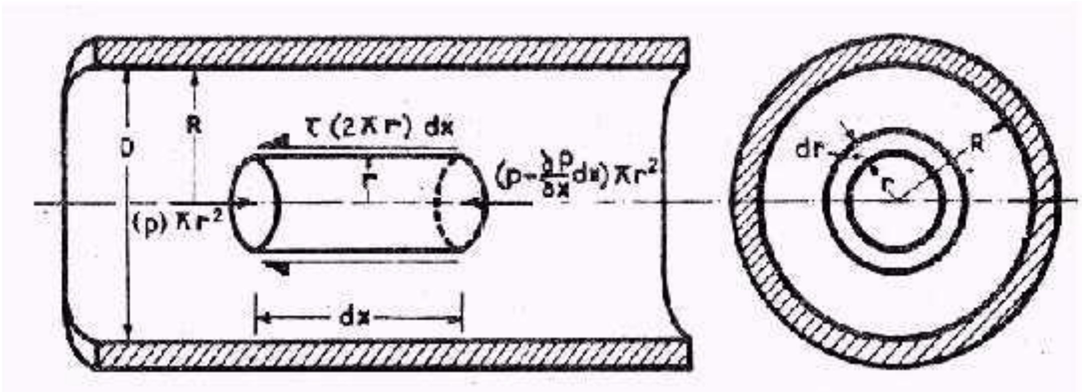
The thickness boundary layer increases with distance from the leading edge in the down-stream direction. Due to increases in thickness of boundary layer, the laminar boundary layer becomes unstable and the motion of the fluid is disturbed. It leads to a transition from laminar to turbulent boundary layer.

2.2.3 BOUNDARY LAYER GROWTH OVER A FLAT PLATE

Consider a continuous flow of fluid along the surface of a thin flat plate with its sharp leading edge set parallel to the flow direction as shown in figure 2.7.The fluid approaches the plate with uniform velocity U known as *free stream velocity* at the leading edge. As soon as the fluid comes in contact the leading edge of the plate,its velocity is reduced to zero as the fluid particles adhere to the plate boundary thereby satisfying no-slip condition.

2.3 FLOW THROUGH CIRCULAR PIPES-HAGEN POISEUILLE'S EQUATION

Due to viscosity of the flowing fluid in a laminar flow,some losses of head take place.The equation which gives us the value of loss of head due to viscosity in a laminar flow is known as Hagen-Poiseuille's law.



$p1-p2=32\mu UL/D^2$

$=128\mu QL/\pi D4$

This equation is called as Hargen-Poiseuille equation for laminar flow in the circular pipes.

2.4 DARCY'S EQUATION FOR LOSS OF HEAD DUE TO FRICTION IN PIPE

A pipe is a closed conduit through which the fluid flows under pressure.When the fluid flows through the piping system,some of the potential energy is lost due to friction.

$hf=4fLv^2/2gD$

2.5 MOODY'S DIAGRAM

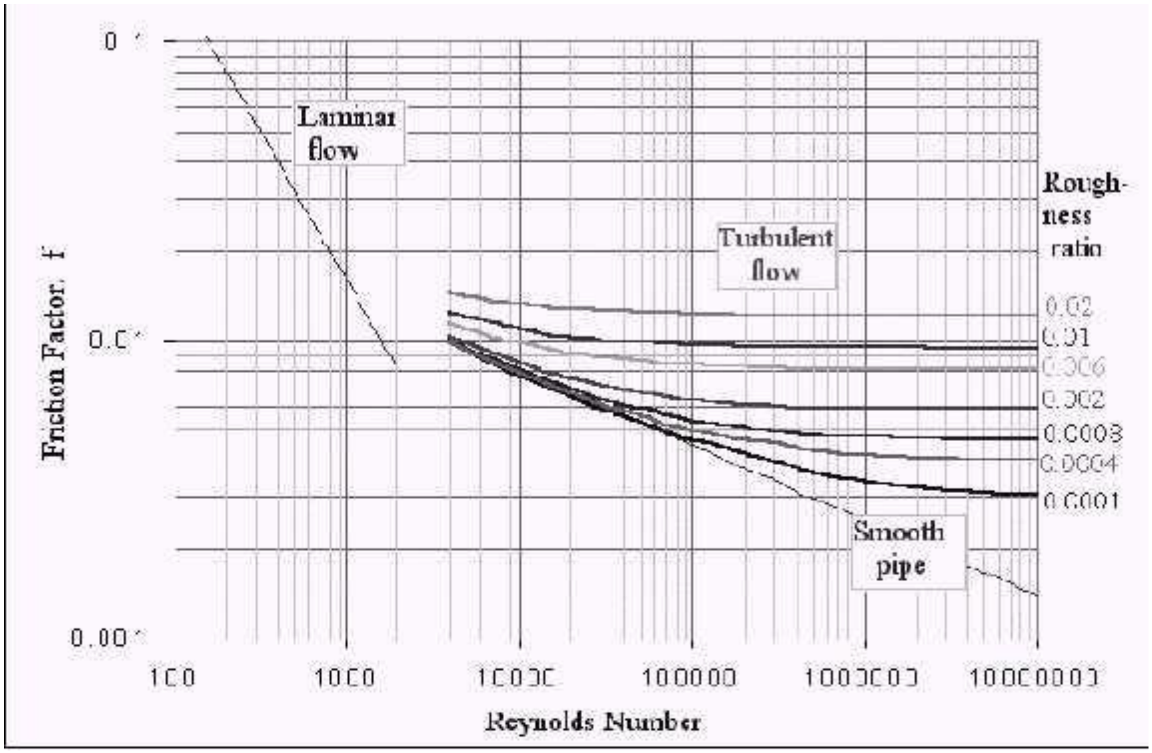
Moody's diagram is plotted between various values of *friction factor(f)*,*Reynolds number(*Re) and *relative roughness*(R/K) as shown in figure 2.6.For any turbulent flow problem,the values of friction factor(f) can therefore be determined from Moody's diagram,if the numerical values of R/K for the pipe and Re of flow are known.

The Moody's diagram has plotted from the equation

$1/\sqrt{f}-2.0 \log10(R/K)=1.74-2.0 \log10[1+18.7/(R/K/Re/ f)]$

Where,R/K=relative roughness

f=friction factor and Re=Reynolds number.



2.6 CLASSIFICATION OF BOUNDARY LAYER THICKNESS

1. Displacements thickness(δ*)
2. Momentum thickness(θ)
3. Energy thickness(δe)

2.7 BOUNDARY LAYER SEPARATION

The boundary layer leaves the surface and gets separated from it. This phenomenon is known as *boundary layer separation.*

2.8 LOSSESS IN PIPES

When a fluid flowing through a pipe, certain resistance is offered to the flowing fluid, it results in causing a loss of energy.

The loss is classified as:

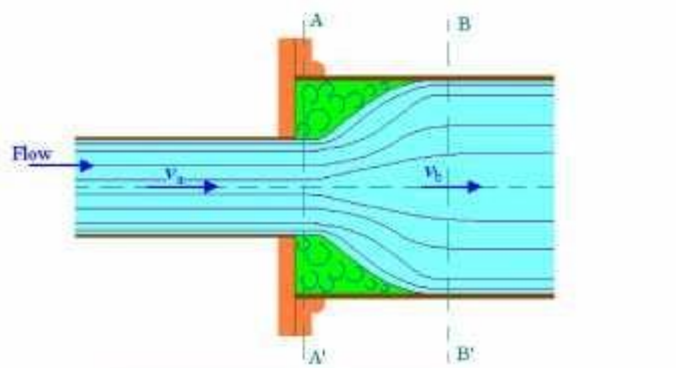
1. Major losses
2. Minor losses

2.8.1 Major Losses in Pipe Flow

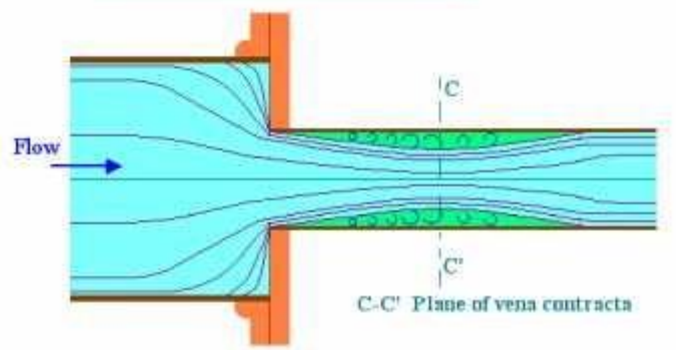
The major loss of energy is caused by friction in pipe. It may be computed by Darcy-weisbach equation.

Minor Losses in Pipe Flow

The loss of energy caused on account of the change in velocity of flowing fluid is called minor loss of energy.



Flow at sudden enlargement of cross section



Flow at sudden contraction of cross section

2.9 FLOW THOUGH PIPES IN SERIES AND PARALLEL

Pipes in Series

The pipes of different diamers and lengths which are connected with one another to form a single pipeline.

Pipes in Parallel

When a main pipeline divides into two or more parallel pipes which again join together to form a single pipe and continuous as a main line

GLOSSARY

HGL –Hydraulic gradient line
TEL – Total energy line.

Applications

1. To find out friction factor in the flow through pipe.
2. To find out the losses in losses in the pipes.

REVIEW QUESTIONS

PART A

1. Mention the general characteristics of laminar flow.
2. Write down the Navier-stokes equation.
3. Write down the Hagen-Poiseuille equation for laminar flow.
4. What are energy lines and hydraulic gradient lines?
5. What is a siphon? What is its application?
6. What is hydraulic Mean Depth or hydraulic radius?
7. Write the Darcy weishbach and Chezy's formulas.
8. Where the Darcy weishbach and Chezy's formulas are used?
9. What are the losses experienced by fluid when it is passing through a pipe?
10. Write the equation of loss of energy due to sudden enlargement.
11. What do you mean by flow through parallel pipes?
12. What is boundary layer?

PART-B

1. (a) Derive an expression for the velocity distribution for viscous flow through a circular pipe.
   (b) A main pipe divides into two parallel pipes, which again forms one pipe. The length and diameter for the first parallel pipe are 2000m and 1m respectively, while the length and diameter of second parallel pipe are 2000 and 0.8 m respectively. Find the rate of flow in each parallel pipe, if total flow in the main is 3 m^3/s. The coefficient of friction for each parallel pipe is same and equal to 0.005.

2. (a)Two pipes of 15 cm and 30 cm diameters are laid in parallel to pass a total discharge of 100 liters/ second. Each pipe is 250 m long. Determine discharge through each pipe. Now these pipes are connected in series to connect two tanks 500 m apart, to carry same total discharge. Determine water level difference between the tanks. Neglect minor losses in both cases, f=0.02 fn both pipes.
   (b) A pipe line carrying oil of specific gravity 0.85, changes in diameter from 350 mm at position 1 to 550 mm diameter to a position 2, which is at 6 m at a higher level. If the pressure at position 1 and 2 are taken as 20 N/cm2 and 15 N/ cm2 respectively and discharge through the pipe is 0.2 m^3/s. determine the loss of head.

3. Obtain an expression for Hagen- Poisulle flow. Deduce the condition of maximum velocity.

4. A flat plate 1.5 m X 1.5 m moves at 50 km / h in a stationary air density 1.15 kg/ m^3. If The coefficient of drag and lift are 0.15 and 0.75 respectively, determine (i) the lift force (ii) the drag force (iii) the resultant force and (iv) the power required to set the plate in motion.

5 (a). The rate of flow of water through a horizontal pipe is 0.3 m^3/s. The diameter of the pipe is suddenly enlarged from 25 cm to 50 cm. The pressure intensity in the smaller pipe is $14 N/m^2$.

Determine (i) Loss of head due to sudden enlargement. (ii) Pressure intensity in the large pipe and (iii) Power lost due to enlargement.

(b) Water is flowing through a tapering pipe of length 200 m having diameters 500 mm at the upper end and 250 mm at the lower end, the pipe has a slope of 1 in 40. The rate of flow through the pipe is 250 lit/ sec. the pressure at the lower end and the upper end are 20 N/cm^2 and 10 N/cm^2 respectively. Find the loss of head and direction of flow.

6. A horizontal pipe of 400 mm diameter is suddenly contracted to a diameter of 200 mm. The pressure intensities in the large and small pipe is given as 15 N/cm^2 and 10 N/cm^2 respectively. Find the loss of head due to contraction, if Cc=0.62, determine also the rate of flow of water.

7. Determine the length of an equivalent pipe of diameter 20 cm and friction factor 0.02 for a given pipe system discharging $0.1 m^3/s$. The pipe system consists of the following:
   (i) A 10 m line of 20 cm dia with f=0.03
   (ii) Three 90° bend, k=0.5 for each
   (iii) Two sudden expansion of diameter 20 to 30 cm
   (iv) A 15 m line of 30 cm diameter with f=0.025 and
   (v) A global valve, fully open, k=10.

UNIT 4 DIMENSIONAL ANALYSIS

DIMENSIONAL ANALYSIS

PRE REQUEST DISCUSSION

Unit III deals with dimensional analysis,models and similitude,and application of dimensionless parameters.

Many important engineering problems cannot be solved completely by theoretical or mathematical methods. Problems of this type are especially common in fluid-flow, heat-flow, and diffusional operations. One method of attacking a problem for which no mathematical equation can be derived is that of empirical experimentations.

For example, the pressure loss from friction in a long, round, straight, smooth pipe depends on all these variables: the length and diameter of the pipe, the flow rate of the liquid, and the density and viscosity of the liquid. If any one of these variables is changed, the pressure drop also changes. The empirical method of obtaining an equation relating these factors to pressure drop requires that the effect of each separate variable be determined in turn by systematically varying that variable while keep all others constant. The procedure is laborious, and is difficult to organize or correlate the results so obtained into a useful relationship for calculations.

There exists a method intermediate between formal mathematical development and a completely empirical study. It is based on the fact that if a theoretical equation does exist among the variables affecting a physical process, that equation must be dimensionally homogeneous. Because of this requirement it is possible to group many factors into a smaller number of dimensionless groups of variables. The groups themselves rather than the separate factors appear in the final equation.

Concepts

Dimensional analysis drastically simplifies the task of fitting experimental data to design equations where a completely mathematical treatment is not possible; it is also useful in checking the consistency of the units in equations, in converting units, and in the scale-up of data obtained in physical models to predict the performance of full-scale model. The method is based on the concept of dimension and the use of *dimensional formulas*.

Dimensional analysis does not yield a numerical equation, and experiment is required to complete the solution of the problem. The result of a dimensional analysis is valuable in pointing a way to correlations of experimental data suitable for engineering use.

3.1 METHODS OF DIMENSIONAL ANALYSIS

If the number of variables involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods.

1. Rayleigh’s method

2. Buckingham’s π theorem

3.1.1 Rayleigh’s method

This method is used for determining the expression for a variable which depends upon maximum three or four variables only. If the number of independent variables becomes more than four then it is very difficult to find the expression for the dependent variable.

3.1.2 Buckingham’s π theorem.

If there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contain m fundamental dimensions (M, L, T), then the variables are arranged into (n-m) dimensionless numbers. Each term is called Buckingham’s π theorem.

Applications

- It is used to justify the dependency of one variable with the other.
- Usually this type of situation occurs in structures and hydraulic machines.
- To solve this problem efficiently, an excellent tool is identified called dimensional analysis.

Geometrical		
Angle	Arc/radius (a ratio)	$[M^0L^0T$
Length	Indludes all linear measurments	$[L]$
Area	Length × Length	$[L^2]$
Volume	Area × Length	$[L^3]$
First moment of area	Area × Length	$[L^3]$
Second moment of area	Area × Length²	$[L^4]$
Strain	Extension/Length	$[L^0]$
Kinematic		
Time	–	$[T]$
Velocity, linear	Distance/Time	$[LT^{-1}]$
Pressure intensity	Force/Area	$[ML^{-1}T^{-2}]$
Stress	Force/Area	$[ML^{-1}T^{-2}]$
Elastic modulus	Stress/strain	$[ML^{-1}T^{-2}]$
Impulse	Force × Time	$[MLT^{-1}]$
Mass moment of inertia	Mass × Length × Length	$[ML^2]$
Momentum, linear	Mass × Linear velocity	$[MLT^{-1}]$
Momentum, angular	MI × Angular velocity	$[ML^2T^{-1}]$
Work, energy	Force × Distance	$[ML^2T^{-2}]$
Power	Work/Time	$[ML^2T^{-3}]$
Moment of a force	Force × Distance	$[ML^2T^{-2}]$
Viscosity, dynamic	Shear stress/Velocity gradient	$[ML^{-1}T^{-1}]$
Viscosity, kinematic	Dynamic viscosity/Mass density	$[L^2T^{-1}]$
Kinematic		
Acceleratoin, linear	Linear velocity/Time	$[LT^{-2}]$
Velocity, angular	Angle/Time	$[T^{-1}]$
Acceleration, angular	Angular velocity/Time	$[T^{-2}]$
Volume rate of discharge	Volume/Time	$[L^3T^{-1}]$
Dynamic		
Mass	Force/Acceleration	$[M]$
Force	Mass × Acceleration	$[MLT^{-2}]$
Weight	Force	$[MLT^{-2}]$
Mass density	Mass/volume	$[ML^{-3}]$
Specific weight	Weight/volume	$[ML^{-2}T^{-2}]$
Specific gravity	Density/Density of water	$[M^0L^0T^0]$

3.2 SMILITUDE –TYPES OF SIMILARITIES

Similitude is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype are completely similar. Three types of similarities must exist between the model and prototype.

Concepts

Whenever it is necessary to perform tests on a model to obtain information that cannot be obtained by analytical means alone, the rules of similitude must be applied. *Similitude* is the theory and art of predicting prototype performance from model observations

1. Geometric similarity refers to linear dimensions. Two vessels of different sizes are geometrically similar if the ratios of the corresponding dimensions on the two scales are the same. If photographs of two vessels are completely super-impossible, they are geometrically similar.

2.Kinematic similarity refers to motion and requires geometric similarity and the same ratio of velocities for the corresponding positions in the vessels.

3.Dynamic similarity concerns forces and requires all force ratios for corresponding positions to be equal in kinematically similar vessels.

SIGNIFICANCE

The requirement for similitude of flow between model and prototype is that the significant dimensionless parameters must be equal for model and prototype

3.3 DIMENSIONLESS PARAMETERS

Since the inertia force is always present in a fluid flow, its ratio with each of the other forces provides a dimensionless number.

1. Reynold’s number
2. Froud’s number
3. Euler’s number
4. Weber’s number
5. Mach’s number

Applications of dimensionless parameters

1. Reynold’s model law
2. Froud’s model law
3. Euler’s model law
4. Weber’s model law
5. Mach’s model law

Concepts

Dimensionless Number	***Symbol***	***Formula***	***Numerator***	***Denominator***	***Importance***
Reynolds number	NRe	$Dv\rho/\mu$	Inertial force	Viscous force	Fluid flow involving viscous and inertial forces
Froude number	N_{Fr}	u^2/gD	Inertial force	Gravitational force	Fluid flow with free surface
Weber number	N_{We}	$u^2\rho D/\sigma$	Inertial force	Surface force	Fluid flow with interfacial forces
Mach number	N_{Ma}	u/c	Local velocity	Sonic velocity	Gas flow at high velocity
Drag coefficient	C_D	$F_D/(\rho u^2/2)$	Total drag force	Inertial force	Flow around solid bodies
Friction factor	f	$\tau_w/(\rho u^2/2)$	Shear force	Inertial force	Flow though closed conduits
Pressure coefficient	C_P	$\Delta p/(\rho u^2/2)$	Pressure force	Inertial force	Flow though closed conduits. Pressure drop estimation

Generally, type 1 can be solved directly, where as types 2 and 3 require simple trial and error.

Three fundamental problems which are commonly encountered in pipe-flow calculations:
Constants: rho, mu, g, L

1. Given D, and v or Q, compute the pressure drop. (pressure-drop problem)
2. Given D, delP, compute velocity or flow rate (flow-rate problem)
3. Given Q, delP, compute the diameter D of the pipe (sizing problem)

REVIEW QUESTIONS

1. Define Dimensional Analysis
2. What you meant by fundamental and derived units?
3. Define dimensionally homogeneous equation.
4. What are the methods of dimensional analysis?
5. State Buckingham's Π theorem
6. What you meant by Repeating variables
7. What is dimensionless number?
8. Check the dimensional homogeneity for the equation V=u+at

PART-B

1) Check the dimensional homogeneity for the equation **V = u + ft.**

2) Determine the dimension of the following quantities:
    i) Discharge
    ii) Kinematic viscosity
    iii) Force and
    iv) Specific weight.

3) Find an expression for the drag force on smooth sphere of diameter ***D***, moving with uniform velocity ***v***, in fluid density ρ and dynamic viscosity μ

4) Efficiency η of a fan depends on the density ρ the dynamic viscosity of the fluid μ, the angular velocity ω, diameter ***D*** of the rotor and the discharge ***Q***. Express η in terms of dimensional parameters.

5) The resistance force ***R*** of a supersonic plane during flight can be considered as dependent upon the length of the aircraft ***l***, velocity ***v***, air viscosity μ, air density ρ and bulk modulus of air ***K***. Express the functional relationship between these variables and the resisting force.

6) A partially submerged body is towed in water. The resistance ***R*** to its motion depends on the density ρ, the viscosity μ of water, length ***l*** of the body, velocity ***v*** of

the body and acceleration due to gravity g. Show that the resistance to motion can be expressed in the form

$$R = \rho L^2 v^2 \phi \left[\left(\frac{\mu}{\rho L v}\right) \left(\frac{lg}{v^2}\right)\right].$$

7) The pressure drop Δp in a pipe of diameter D and length l depends on the d ρ aen \tycandtvi μ osity glof puitufle wnce, t. S an velocity v of flow and average height of protuberance Δt. hc ϕ ssure drop can be expressed in the form $\Delta p = \rho v^2 \phi \left[\frac{l}{D}, \frac{\mu}{vD\rho}, \frac{t}{D}\right]$.

8) Find the expression for the drag force on smooth sphere of diameter D moving with uniform velocity v in fluid density ρ and dynamic viscosity μ

9) The efficiency η of a fan depends on the density ρ, the dynamic viscosity μ, angular velocity ω, diameter D of the motor and the discharge Q. Express the efficiency η in terms of dimensional parameters.

10) The pressure difference Δp in a pipe of diameter D and length l due to turbulent flow depends on the velocity v, viscosity μ, density ρ and oughness K. Using Buckingham's π-theorem, obtain an expression for Δp.

UNIT-IV PUMPS

PRE REQUEST DISCUSSION

Basic concepts of rot dynamic machines, velocity triangles for radial flow and axial flow machines, centrifugal pumps, turbines and Positive displacement pumps and rotary pumps its performance curves are discussed in Unit IV.

The liquids used in the chemical industries differ considerably in physical and chemical properties. And it has been necessary to develop a wide variety of pumping equipment.

The two main forms are the *positive displacement type* and *centrifugal pumps*.

the former, the volume of liquid delivered is directly related to the displacement of the piston and therefore increases directly with speed and is not appreciably influenced by the pressure. In this group are the *reciprocating piston pump* and the *rotary gear pump*, both of which are commonly used for delivery against high pressures and where nearly constant delivery rates are required.

The centrifugal type depends on giving the liquid a high kinetic energy which is then converted as efficiently as possible into pressure energy.

4.1 HEAD AND EFFICIANCES

1. Gross head
2. Effective or Net head
3. Water and Bucket power
4. Hydraulic efficiency
5. Mechanical efficiency
6. Volume efficiency
7. Overall efficiency

Concepts

A pump is a device which converts the mechanical energy supplied into hydraulic energy by lifting water to higher levels.

4.2 CENTRIFUGAL PUMP

Working principle

If the mechanical energy is converted into pressure energy by means of centrifugal force acting
on the fluid, the hydraulic machine is called centrifugal pump. The centrifugal pump acts as a reversed of an inward radial flow reaction turbine

4.2.3 Performance Characteristics of Pumps

The fluid quantities involved in all hydraulic machines are the flow rate (Q) and the head (H), whereas the mechanical quantities associated with the machine itself are the power (P), speed (N), size (D) and efficiency (η). Although they are of equal importance, the emphasis placed on certain of these quantities is different for different pumps. The output of a pump running at a given speed is the flow rate delivered by it and the head developed. Thus, a plot of head and flow rate at a given speed forms the fundamental performance characteristic of a pump. In order to achieve this performance, a power input is required which involves efficiency of energy transfer. Thus, it is useful to plot also the power P and the efficiency η against Q.

Over all efficiency of a pump (η) = Fluid power output / Power input to the shaft = ρgHQ / P

Type number or Specific speed of pump, $n_S = NQ^{1/2} / (gH)^{3/4}$ (it is a dimensionless number)

Centrifugal pump Performance

In the volute of the centrifugal pump, the cross section of the liquid path is greater than in the impeller, and in an ideal frictionless pump the drop from the velocity V to the lower velocity is converted according to Bernoulli's equation, to an increased pressure. This is the source of the discharge pressure of a centrifugal pump.

If the speed of the impeller is increased from N_1 to N_2 rpm, the flow rate will increase from Q_1 to Q_2 as per the given formula:

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2}$$

The head developed(H) will be proportional to the square of the quantity discharged, so that

$$\frac{H_1}{H_2} = \frac{Q_1^2}{Q_2^2} = \frac{N_1^2}{N_2^2}$$

The power consumed(W) will be the product of H and Q, and, therefore

$$\frac{W_1}{W_2} = \frac{Q_1^3}{Q_2^3} = \frac{N_1^3}{N_2^3}$$

These relationships, however, form only the roughest guide to the performance of centrifugal pumps.

4.2.4 Characteristic curves

Pump action and the performance of a pump are defined interms of their *characteristic curves*. These curves correlate the capacity of the pump in unit volume per unit time versus discharge or differential pressures. These curves usually supplied by pump manufacturers are for water only.

These curves usually shows the following relationships (for centrifugal pump).

- A plot of capacity versus differential head. The differential head is the difference in pressure between the suction and discharge.
- The pump efficiency as a percentage versus capacity.
- The break horsepower of the pump versus capacity.
- The net poisitive head required by the pump versus capacity. The required NPSH for the pump is a characteristic determined by the manufacturer.

Centrifugal pumps are usually rated on the basis of head and capacity at the point of maximum efficiency.

4.3 RECIPROCATING PUMPS

Working principle

If the mechanical energy is converted into hydraulic energy (or pressure energy) by sucking
the liquid into a cylinder in which a piston is reciprocating (moving backwards and forwards), which
exerts the thrust on the liquid and increases its hydraulic energy (pressure energy), the pump is known as reciprocating pump

Main ports of a reciprocating pump

1.A cylinder with a piston, piston rod, connecting rod and a crank, 2. Suction pipe

3.Delivery pipe, 4. Suction valve and 5.Delivery valve.

Slip of Reciprocating Pump

Slip of a reciprocating pump is defined as the difference between the theoretical discharge and the actual discharge of the pump.

4.3.1Characteristic Curves Of Reciprocatring Pumps

1. According to the water being on contact with one side or both sides of the piston
   (i.) Single acting pump (ii.) Double-acting pump

2. According to the number of cylinders provided
(i.) Single acting pump (ii.) Double-acting pump (iii.) Triple-acting pump

Reciprocating pumps Vs centrifugal pumps

The advantages of reciprocating pumps in general over centrifugal pumps may be summarized as follows:

1. They can be designed for higher heads than centrifugal pumps.
2. They are not subject to air binding, and the suction may be under a pressure less than atmospheric without necessitating special devices for priming.
3. They are more flexible in operation than centrifugal pumps.
4. They operate at nearly constant efficiency over a wide range of flow rates.

The advantages of centrifugal pumps over reciprocating pumps are:

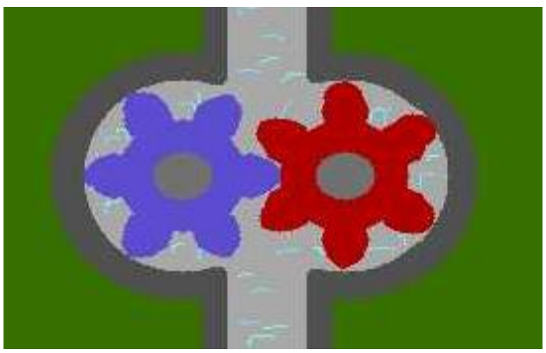
1. The simplest centrifugal pumps are cheaper than the simplest reciprocating pumps.
2. Centrifugal pumps deliver liquid at uniform pressure without shocks or pulsations.

3. They can be directly connected to motor derive without the use of gears or belts.
4. Valves in the discharge line may be completely closed without injuring them.
5. They can handle liquids with large amounts of solids in suspension.

4.4 Rotary Pumps

The rotary pump is good for handling viscous liquids, nut because of the close tolerances needed, it can not be manufactured large enough to compete with centrifugal pumps for coping with very high flow rates.
Rotary pumps are available in a variety of configurations.

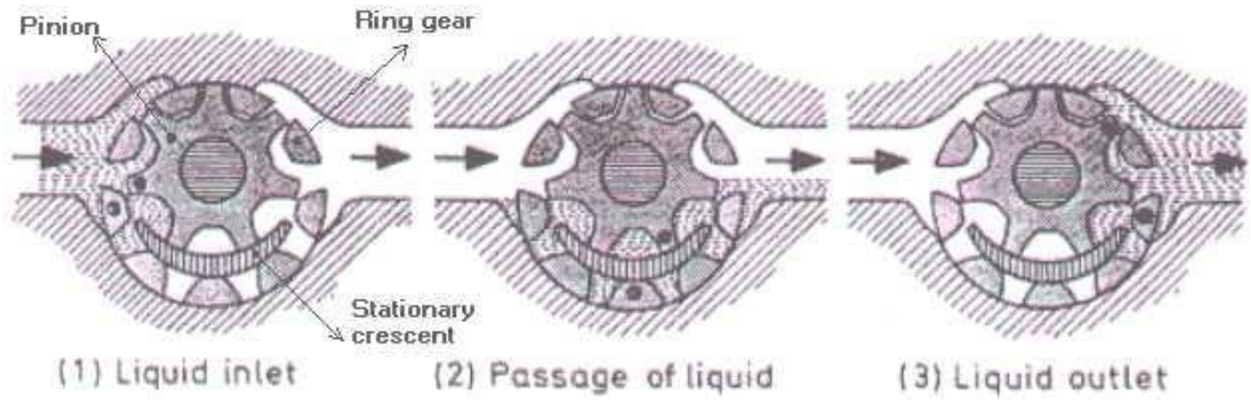
- Double lobe pump
- Trible lobe pumps
- Gear pump
- Gear Pumps
-
- **Spur Gear or External-gear pump**



External-gear pump (called as gear pump) consists essentially of two intermeshing gears which are identical and which are surrounded by a closely fitting casing. One of the gears is driven directly by the prime mover while the other is allowed to rotate freely. The fluid enters the spaces between the teeth and the casing and moves with the teeth along the outer periphery until it reaches the outlet where it is expelled from the pump.

External-gear pumps are used for flow rates up to about 400 m^3/hr working against pressures as high as 170 atm. The volumetric efficiency of gear pumps is in the order of 96 percent at pressures of about 40 atm but decreases as the pressure rises.

4.4.1 Internal-gear Pump

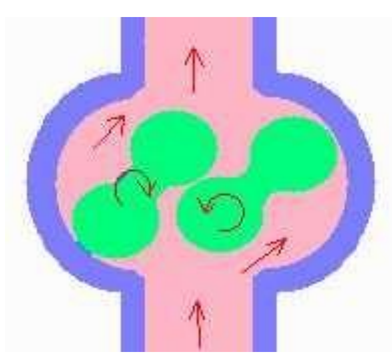


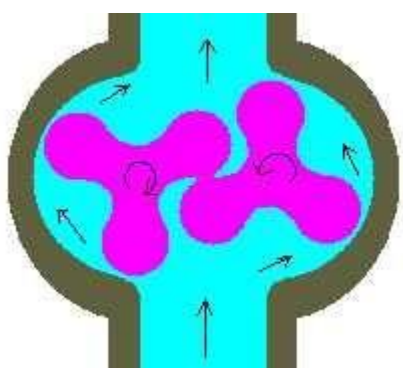
The above figure shows the operation of a internal gear pump. In the internal-gear pump a spur gear, or pinion, meshes with a ring gear with internal teeth. Both gears are inside the casing. The ring gear is coaxial with the inside of the casing, but the pinion, which is externally driven, is mounted eccentrically with respect to the center of the casing. A stationary metal crescent fills the space between the two gears. Liquid is carried from inlet to discharge by both gears, in the spaces between the gear teeth and the crescent.

4.4.2 Lobe pumps

In principle the lobe pump is similar to the external gear pump; liquid flows into the region created as the counter-rotating lobes unmesh. Displacement volumes are formed between the surfaces of each lobe and the casing, and the liquid is displaced by meshing of the lobes. Relatively large displacement volumes enable large solids (nonabrasive) to be handled. They also tend to keep liquid velocities and shear low, making the pump type suitable for high viscosity, shear-sensitive liquids.

Two lobe pump   Three lobe pump





The choice of two or three lobe rotors depends upon solids size, liquid viscosity, and tolerance of flow pulsation. Two lobe handles larger solids and high viscosity but pulsates more. Larger lobe pumps cost 4-5 times a centrifugal pump of equal flow and head.

4.3 Selection of Pumps

The following factors influence the choice of pump for a particular operation:

1. *The quantity of liquid to be handled*: This primarily affects the size of the pump and determines whether it is desirable to use a number of pumps in parallel.
2. *The head against which the liquid is to be pumped*. This will be determined by the difference in pressure, the vertical height of the downstream and upstream reservoirs and by the frictional losses which occur in the delivery line. The suitability of a centrifugal pump and the number of stages required will largely be determined by this factor.
3. *The nature of the liquid to be pumped*. For a given throughput, the viscosity largely determines the frictional losses and hence the power required. The corrosive nature will determine the material of construction both for the pump and the packing. With suspensions, the clearance in the pump must be large compared with the size of the particles.
4. *The nature of power supply*. If the pump is to be driven by an electric motor or internal combustion engine, a high-speed centrifugal or rotary pump will be preferred as it can be coupled directly to the motor.
5. *If the pump is used only intermittently,* corrosion troubles are more likely than with continuous working.

Applications

The handling of liquids which are particularly corrosive or contain abrasive solids in suspension, compressed air is used as the motive force instead of a mechanical pump.

REVIEW QUESTIONS

PART A

1. What is meant by Pump?
2. Mention main components of Centrifugal pump.
3. What is meant by Priming?
4. Define Manometric head.
5. Define Manometric efficiency
6. Define Mechanical efficiency.
7. Define overall efficiency.
8. Give the range of specific speed for low, medium, high speed radial flow.
9. Define speed ratio, flow ratio.
10. Mention main components of Reciprocating pump.
11. Define Slip of reciprocating pump. When the negative slip does occur?

PART-B

1. Write short notes on the following (1) Cavitations in hydraulic machines their causes, effects and remedies. (2) Type of rotary pumps.

2. Draw a neat sketch of centrifugal pump and explain the working principle of the centrifugal pump.

3. Draw a neat sketch of Reciprocating pump and explain the working principle of single acing and double acting Reciprocating pump.

4. A radial flow impeller has a diameter 25 cm and width 7.5 cm at exit. It delivers 120 liters of water per second against a head of 24 m at 1440 rpm. Assuming the vanes block the flow area by 5% and hydraulic efficiency of 0.8, estimate the vane angle at exit. Also calculate the torque exerted on the driving shaft if the mechanical efficiency is 95%.

5. Find the power required to drive a centrifugal pump which to drive a centrifugal pump which delivers 0.04 m3 /s of water to a height of 20 m through a 15 cm diameter pipe and 100 m long. The over all efficiency of the pump is 70% and coefficient of friction is 0.15 in the formula hf=4flv2/2gd.

6. A Centrifugal pump having outer diameter equal to 2 times the inner diameter and running at 1200 rpm works against a total head of 75 m. The Velocity of flow through the impeller is constant and equal to 3 m/s. The vanes are set back at an angle of 30º at out let. If the outer diameter of impeller is 600 mm and width at outlet is 50 mm. Determine (i) Vane angle at inlet (ii) Work done per second on impeller (iii) Manometric efficiency.

7. The diameter and stroke of a single acting reciprocating pump are 200 mm and 400 mm respectively, the pump runs at 60 rpm and lifts 12 liters of water per second through a height of 25 m. The delivery pipe is 20m long and 150mm in diameter. Find (i) Theoretical power required to run the pump. (ii) Percentage of slip. (iii) Acceleration head at the beginning and middle of the delivery stroke.

UNIT V TURBINES

PRE REQUEST DISCUSSION

Hydraulic Machines are defined as those machines which convert either hydraulic energy (energy possessed by water) into mechanical energy (which is further converted into electrical energy) or mechanical energy into hydraulic energy. The hydraulic machines, which convert the hydraulic energy into mechanical energy, are called turbines.

Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy. This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine. Thus the mechanical energy is converted into electrical energy. The electric power which is obtained from the hydraulic energy (energy of water) is known as Hydro- electro power.

In our subject point of view, the following turbines are important and will be discussed one by one.

1. Pelton wheel

2. Francis turbine

3. Kaplan turbine

Concept

Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy. This mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine

FLUID	TYPES OF TURBINE
Water	Hydraulic Turbine
Steam	Steam Turbine
Froen	Vapour Turbine
Gas or air	Gas Turbine
Wind	Wind Mills

5.1 CLASSIFICATION OF HYDRAULIC TURBINES

1. According to the action of the water flowing

2. According to the main direction of flow of water

3. According to the head and quality of water required

4. According to the specific speed

5.2 HEAD AND EFFICIANCES OF PELTON WHEEL

1. Gross head

2. Effective or Net head

3. Water and Bucket power

4. Hydraulic efficiency

5. Mechanical efficiency

6. Volume efficiency

7. Overall efficiency

5.3 IMPULSE TURBINE

In an impulse turbine, all the energy available by water is converted into kinetic energy by passing a nozzle. The high velocity jet coming out of the nozzle then impinges on a series of buckets fixed around the rim of a wheel.

5.4 Tangential Flow Turbine, Radial And Axial Turbines

1. Tangential flow turbine

In a tangential flow turbine, water flows along the tangent to the path of runner. E.g. Pelton wheel

2. Radial flow turbine

In a radial flow turbine, water flows along the radial direction and mainly in the plane normal to the axis of rotation, as it passes through the runner. It may be either inward radial flow type or outward radial flow type.

3. Axial flow turbine

In axial flow turbines, water flows parallel to the axis of the turbine shaft. E.g. kaplan turbine

4. Mixed flow turbine

In a mixed flow turbine, the water enters the blades radiallsy and comes out axially and parallel to the turbine shaft .E.g. Modern Francis turbine.

In our subject point of view, the following turbines are important and will be discussed one by one

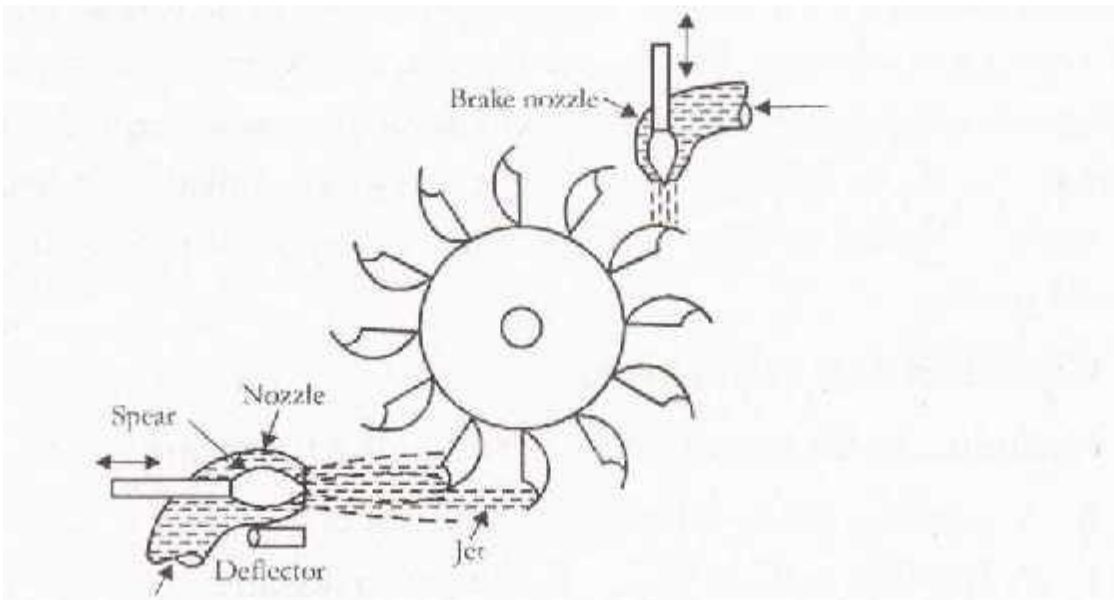
1. Pelton wheel

2. Francis turbine

3. Kaplan turbine

5.5 PELTON WHEEL OR PELTON TURBINE

The Pelton wheel is a tangential flow impulse turbine and now in common use. Leston A Pelton, an American engineer during 1880,develops this turbines. A pelton wheel consists of following main parts.



1. Penstock
2. Spear and nozzle
3. Runner with buckets
4. Brake nozzle
5. Outer casing
6. Governing mechanism

5.5.1 VELOCITY TRIANGLES, WORKDONE, EFFICIENCY OF PELTON WHEEL INLET AND OUTLET VECTOR DIAGRAMS

Let V = Velocity of the jet
u = Velocity of the vane (cups) at the impact point u
= DN/ 60

where D = Diameter of the wheel corresponding to the impact point
= Pitch circle diameter.
At inlet the shape of the vane is such that the direction of motion of the jet and the vane is the same.
i.e., Ȑ = 0, ș = 0
Relative velocity at inlet $V_r = V - u$

The Pelton wheel being a parallel flow turbine the peripheral velocities at and outlet are equal.

i.e., $u = u_1$

If the frictional resistance along the vanes is ignored then relative at inlet and outlet are also equal.

i.e., $V_r = V_{r1}$

Let θ = Outlet vane angle

Velocity of whirl at inlet

$$= V_w = V$$

Velocity of whirl at outlet

$$= V_{w1} = u_1 - V_{r1}\cos\phi$$

But $u_1 = u$

$$V_{r1} = V_r = V - u$$

$$\therefore \quad V_{w1} = u - (V - u)\cos\phi$$

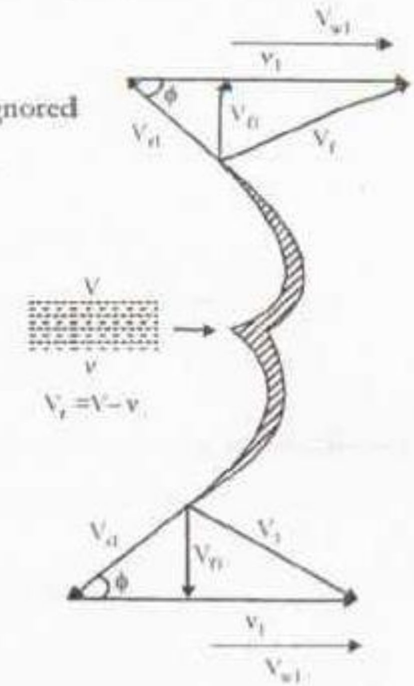


Figure: 4.8

$\therefore$ Force exerted per N of water

$$= \frac{1}{g}\left[V_w - V_{w1}\right] = \frac{1}{g}\left[V - u + (V - u)\cos\phi\right]$$

$$= \frac{1}{g}(V - u)(1 + \cos\phi)$$

$\therefore$ Work done per second per N of water

$$U = \frac{1}{g}(V - u)(1 + \cos\phi)u$$

$$U = \left[\frac{1 + \cos\phi}{g}\right]u(V - u)$$

Hydraulic efficiency

This is the ratio of the work done per second per head at inlet to the turbine.

Energy head at inlet = $V^2/2g$

∴ Hydraulic efficiency

$$\eta_h = \frac{U}{\left(\frac{V^2}{2g}\right)}$$

$$\eta_h = \frac{\left(\frac{1+\cos\phi}{g}\right)u(V-u)}{\frac{V^2}{2g}}$$

Condition for maximum hydraulic efficiency

For a given jet velocity for efficiency to be maximum, word done should be maximum

Work done per second per N of water

$$= U = \left(\frac{1+\cos\phi}{g}\right)u(V-u)$$

For U to be maximum, $\frac{dU}{du} = 0$

$$\therefore \quad \frac{dU}{du} = \left(\frac{1+\cos\phi}{g}\right)(V-2u) = 0$$

$$\therefore \quad V - 2u = 0$$

$$\therefore \quad u = \frac{V}{2}$$

Hence for the condition of maximum hydraulic efficiency, the peripheral speed of the turbine should reach one half the jet speed.

$$\therefore \quad U_{max} = \left[\frac{1+\cos\phi}{g}\right]\frac{V}{2}\left(V - \frac{V}{2}\right)$$

$$U_{max} = \frac{V^2}{4g}\left(1+\cos\phi\right)$$

Maximum hydraulic efficiency

$$\eta_h(max) = \frac{\frac{V^2}{4g}\left(1+\cos\phi\right)}{\frac{V^2}{2g}}$$

$$\eta_h(max) = \frac{1+\cos\phi}{2}$$

5.6 SPECIFIC SPEED

[The speed of any water turbine is represented by N rpm. A turbine has speed, known as specific speed and is represented by N
‘ Specific speed of a water turbine in the speed at which a geometrically similar turbine would run if producing unit power (1 kW) and working under a net head of 1 m. Such a turbine would be an imaginary one and is called specific turbine.

Expression *for specific speed*

Let us consider the case of a reaction turbine.

As, $Q = \pi DB \times V_f$

and $u = \frac{\pi DN}{60}$

or $D = \frac{60u}{\pi N}$

$\therefore \quad D \propto \frac{u}{N}$

Again $\frac{u}{\sqrt{2gH}}$ = Speed ratio = constant for a turbine.

or $$u = \text{constant} \times \sqrt{2gH}$$

or $$= u \infty \sqrt{H}$$

$\therefore$ $D \dfrac{\sqrt{H}}{N}$, from equation (4.9) and (4.10)

Now, for a turbine, $B \infty D$

or $B \infty \dfrac{\sqrt{H}}{N}$ [using equation (4.11)

Lastly $\dfrac{u}{\sqrt{2g\,H}}$ = Flow ratio = constant for turbine

or $V_f \infty \sqrt{H}$

Now substituting the value of D, B and and V_f in equation

Hence, $$Q \infty \frac{\sqrt{H}}{N} \times \frac{\sqrt{H}}{N} \times \sqrt{H}$$

or $$Q = \frac{H^{3/2}}{N^2}$$

Also, weight of water $W \infty Q$

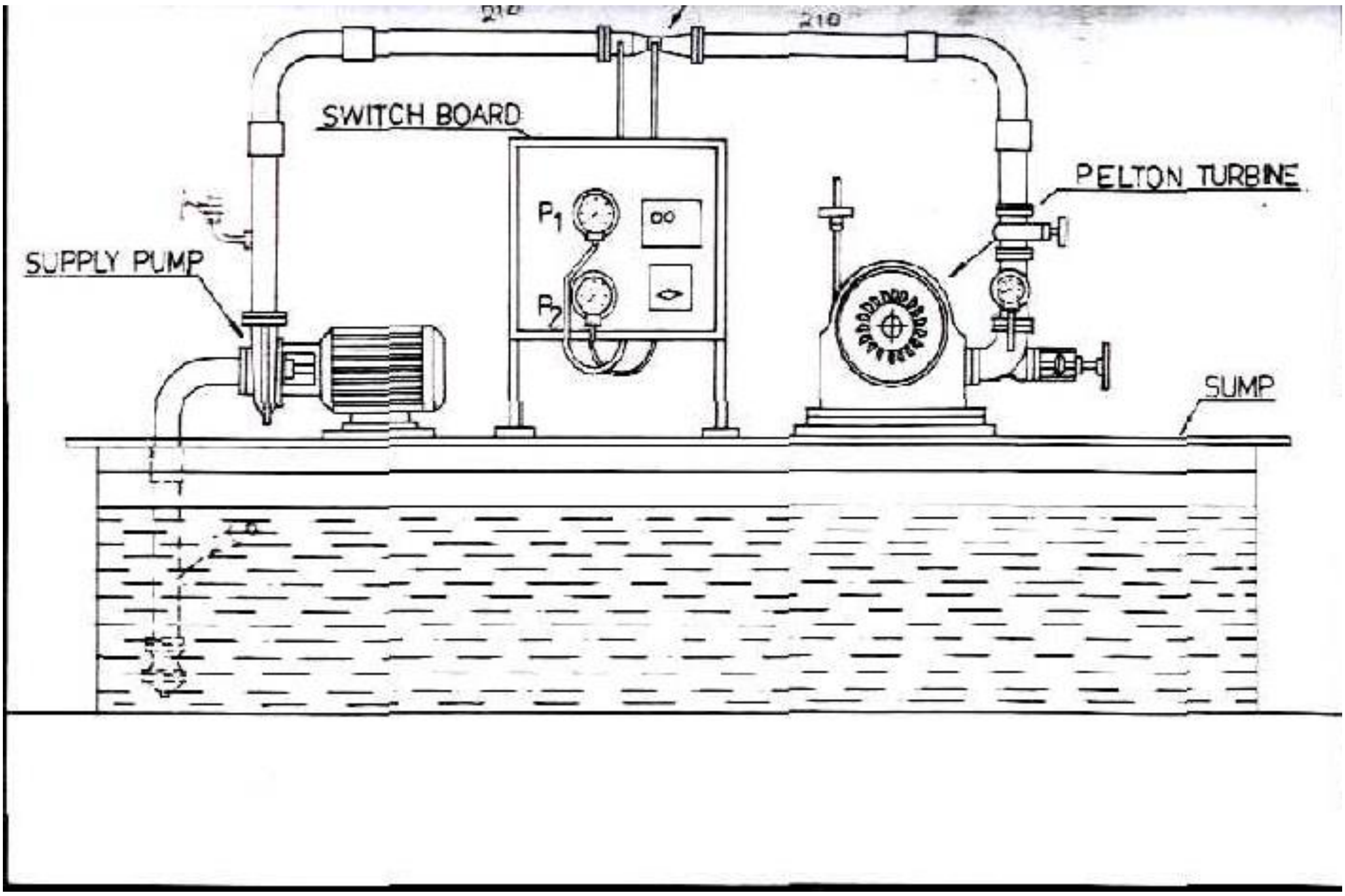
As $W = wQ$

Hence, $$W \infty \frac{H^{3/2}}{N^2}$$

Again $$P = \frac{W.H}{75} \times \eta_0$$

$\therefore$ $P \infty W.H$

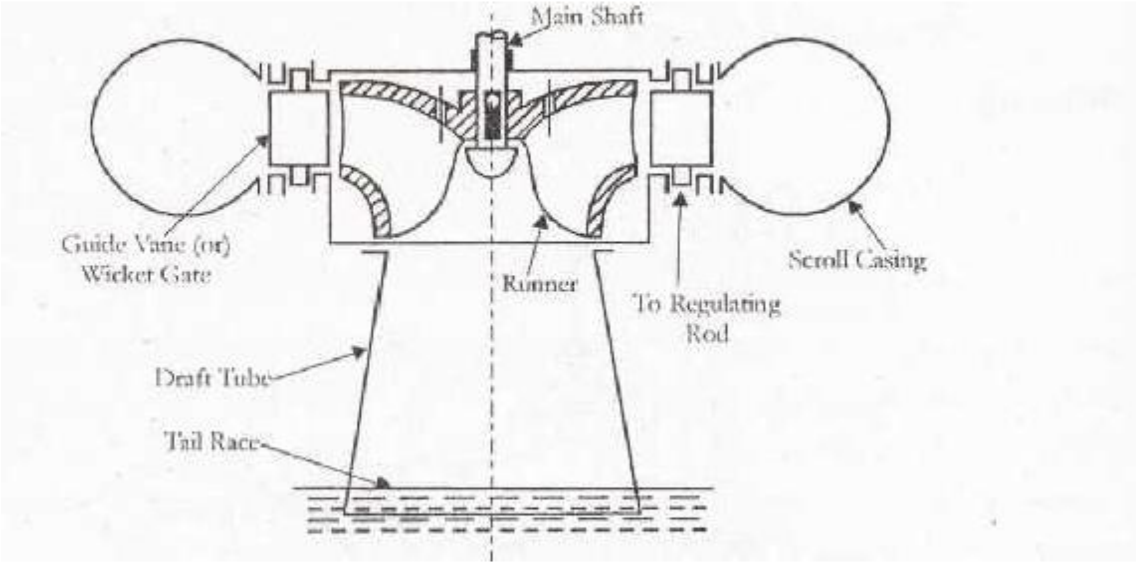
or $$P \infty \frac{\sqrt{H^{3/2}}}{N^2} \times H$$

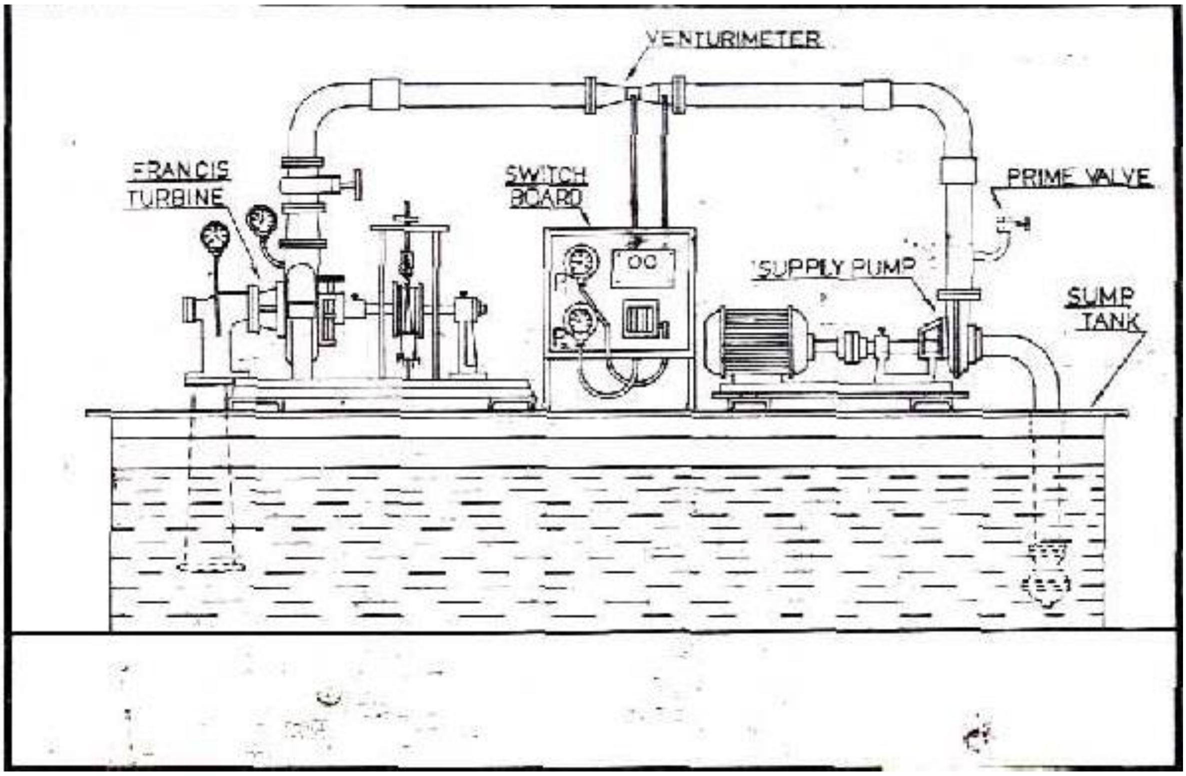


5.7 FRANCIS TURBINE

Francis turbine is an inward flow reaction turbine. It is developed by the American engineer James B. Francis. In the earlier stages, Francis turbine had a purely radial floe runner. But the modern Francis turbine is a mixed flow reaction turbine in which the water enters the runner radially at its outer periphery and leaves axially at its centre. This arrangement provides larger discharge area with prescribed diameter of the runner. The main parts such as

1. Penstock
2. Scroll or Spiral Casing
3. Speed ring or Stay ring
4. Guide vanes or Wickets gates
5. Runner and runner blades
6. Draft tube


Main Shaft
Guide Vane (or)
Wicket Gate
Runner
To Regulating
Rod
Scroll Casing
Draft Tube
Tail Race


VENTURIMETER
FRANCIS
TURBINE
SWITCH
BOARD
PRIME VALVE
SUPPLY PUMP
SUMP
TANK

5.8 KAPLAN TURBINE

A Kaplan turbine is an axial flow reaction turbine which was developed by Austrian engineer V. Kaplan. It is suitable for relatively low heads. Hence, it requires a large quantity of water to develop large power. The main parts of Kaplan turbine, they are

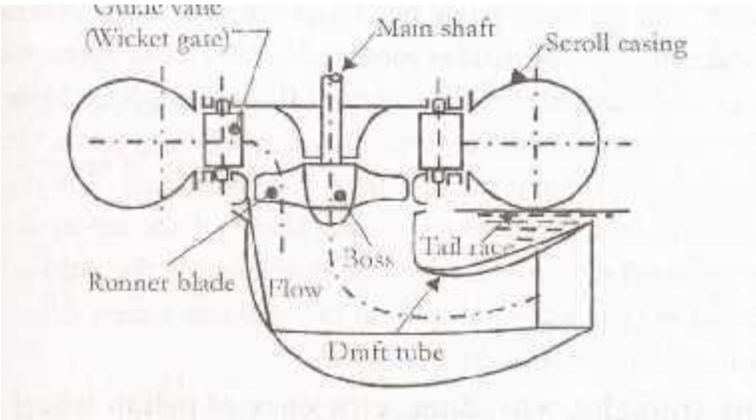
1. Scroll casing

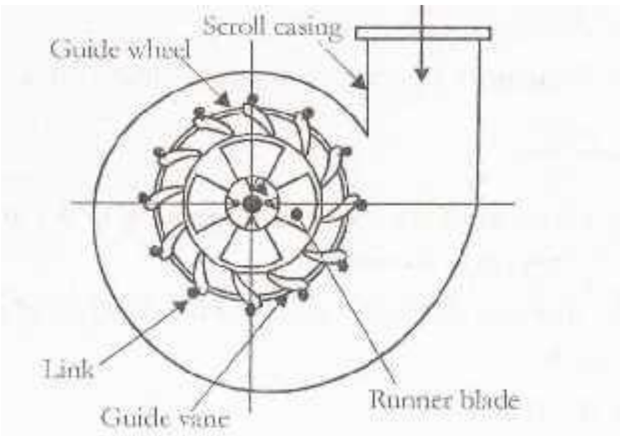
2. Stay ring

3. Guide vanes

4. Runner

5. Draft tube





5.9 PERFORMANCE OF TURBINES

Turbines are often required to work under varying conditions of head, speed, output and gate opening. In order to predict their behavior, it is essential to study the performance of the turbines under the varying conditions. The concept of unit quantities and specific quantities are required to

- The behavior of a turbine is predicted working under different conditions.
- Comparison is made between the performance of turbine of same type but of different sizes.

- ❖ The performance of turbine is compared with different types.

5.10 DRAFT TUBE

The pressure at the exit of the runner of a reaction turbine is generally less than atmospheric pressure. Thus the water at the exit of the runner cannot be directly discharged to the tail race. A pipe o gradually increasing area is used for discharging water form the exit of the turbine to the tail race. This pipe of gradually increasing area is called a draft tube.

5.11 SPECIFIC SPEED

Homologus units are required in governing dimensionless groups to use scaled models in designing turbomachines, based geometric similitude.

Specific speed is the speed of a geometrically similar turbine, which will develop unit power when working under a unit head. The specific speed is used in comparing the different types of turbines as every type of turbine has different specific speed. In S.I. units, unit power is taken as one Kw and unit as one meter.

5.12 GOVERNING OF TURBINES

All the modern hydraulic turbines are directly coupled to the electric generators. The generators are always required to run at constant speed irrespective of the variations in the load. It is usually done by regulating the quantity of water flowing through the runner in accordance with the variations in the load. Such an operation of regulation of speed of turbine runner is known as governing of turbine and is usually done automatically by means of a governor.

Applications

1. To produce the power by water.

GLOSSARY

HP –Horse power
KW- Kilo watts

REVIEW QUESTIONS

1.Define hydraulic machines.
2. Give example for a low head, medium head and high head turbine.
3. What is impulse turbine? Give example.
4. What is reaction turbine? Give example.
5. What is axial flow turbine?
6. What is the function of spear and nozzle?
9. Define gross head and net or effective head.
7.Define hydraulic efficiency.
8.Define unit speed of turbine.

9. Define specific speed of turbine.
10. Give the range of specific speed values of Kaplan , Francis turbine and pelton wheels
11. Define unit discharge.
12. Define unit power.
13. What is a draft tube? In which type of turbine it is mostly used?
14. Write the function of draft tube in turbine outlet.

PART B

1. Obtain en expression for the work done per second by water on the runner of a pelton wheel. Hence derive an expression for maximum efficiency of the pelton wheel giving the relationship between the jet speed and bucket speed.

2. (a) A pelton wheel is having a mean bucket diameter of 1 m and is running at 1000 rpm. The net head on the pelton wheel is 700 m. If the side clearance angle is 15º and discharge through nozzle is 0.1 m^3/s, find (1) power available at nozzle and (2) hydraulic efficiency of the turbine. Take Cv=1 (b) A turbine is to operate under a head of 25 m at 200 rpm. The discharge is 9 m^3/s. If the efficiency is 90% determine, Specific speed of the machine, Power generated and type of turbine.

3. A pelton turbine is required to develop 9000 KW when working under a head of 300 m the impeller may rotate at 500 rpm. Assuming a jet ratio of 10 And an overall efficiency of 85% calculate (1) Quantity of water required. (2) Diameter of the wheel (3) Number of jets (4) Number and size of the bucket vanes on the runner.

4. An Outward flow reaction turbine has internal and external diameters of the runner as 0.5 m and 1.0 m respectively. The turbine is running at 250 rpm and rate of flow of water through the turbine is 8 m^3/s. The width of the runner is constant at inlet and out let and is equal to 30 cm. The head on the turbine is 10 m and discharge at outlet6 is radial, determine (1) Vane angle at inlet and outlet. (2) Velocity of flow at inlet and outlet.

5. The Nozzle of a pelton Wheel gives a jet of 9 cm diameter and velocity 75 m/s. Coefficient of velocity is 0.978. The pitch circle diameter is 1.5 m and the deflection angle of the bucket is 170º. The wheel velocity is 0.46 times the jet velocity. Estimate the speed of the pelton wheel turbine in rpm, theoretical power developed and also the efficiency of the turbine.

6. (a)A turbine is to operate a head of a 25 m at 200 rpm; the available discharge is 9 m^3/s assuming an efficiency of 90%. Determine (1) Specific speed (2) Power generated (3) Performance under a head of 20 m (4) The type of turbine. **)** (b) A vertical reaction turbine under 6m head at 400 rpm the area and diameter of runner at inlet are 0.7 m^2 and 1m respective the absolute and relative velocities of fluid entering are 15ºand 60º to the tangential direction. Calculate hydraulic efficiency.

7. A Francis turbine has an inlet diameter of 2.0 m and an outlet diameter of 1.2m. The width of the blades is constant at 0.2 m. The runner rotates at a speed of 250 rpm with a discharge of 8 m^3/s .The vanes are radial at the inlet and the discharge is radially outwards at the outlet. Calculate the angle of guide vane at inlet and blade angle at the outlet.

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2.Bansal, R.K., Fluid Mechanics and Hydraulics Machines,

3. G.K.Vijayaraghavan Fluid Mechanics and Machinery

QUESTION BANK

UNIT- I
FLUID PROPERTIES AND FLOW CHARACTERISTICS
PART – A

1. Define fluids.

Fluid may be defined as a substance which is capable of flowing. It has no definite shape of its own, but confirms to the shape of the containing vessel.

2. What are the properties of ideal fluid?

Ideal fluids have following properties

i)It is incompressible

ii) It has zero viscosity

iii) Shear force is zero

3. What are the properties of real fluid?

Real fluids have following properties

i)It is compressible

ii) They are viscous in nature

iii) Shear force exists always in such fluids.

4. Explain the Density

Density or mass density is defined as the ratio of the mass of the fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by the symbol (ρ).

$$\text{Density} = \frac{\text{Mass of the fluid (kg)}}{\text{Volume of the fluid (m3)}}$$

5. Explain the Specific weight or weight density

Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume. Thus weight per uint volume of a fluid is called weight density and is denoted by the symbol (W).

$$(W) = \frac{\text{Weight of the fluid}}{\text{Volume of fluid}} = \frac{\text{Mass x Acceleration due to gravity}}{\text{Volume of fluid}}$$

$$W = pg$$

6. Explain the Specific volume

Specific volume of a fluid is defined as the volume of the fluid occupied by a unit Mass or volume per unit mass of a fluid is called specific volume.

$$\text{Specific volume} = \frac{\text{Volume}}{\text{Mass}} = \frac{\text{m3}}{\text{kg}} = \frac{1}{p}$$

7. Explain the Specific gravity

Specific gravity is defined as the ratio of weight density of a fluid to the weight density of a standard fluid. For liquid, standard fluid is water and for gases, it is air.

$$\text{Specific gravity} = \frac{\text{Weight density of any liquid or gas}}{\text{Weight density of standard liquid or gas}}$$

8. Define Viscosity.

It is defined as the property of a liquid due to which it offers resistance to the movement of one layer of liquid over another adjacent layer.

9. Define kinematic viscosity.

It is defined as the ratio of dynamic viscosity to mass density. (m^2/sec)

10. Define Relative or Specific viscosity.

It is the ratio of dynamic viscosity of fluid to dynamic viscosity of water at 20°C.

11. State Newton's law of viscosity and give examples.

Newton's law states that the shear stress ( ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called co-efficient of viscosity.

$$r = \mu \frac{du}{dy}$$

12. Give the importance of viscosity on fluid motion and its effect on temperature.

Viscosity is the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid. The viscosity is an important property which offers the fluid motion.

The viscosity of liquid decreases with increase in temperature and for gas it Increases with increase in temperature.

13. Explain the Newtonian fluid

The fluid which obeys the Newton's law of viscosity i.e., the shear stress is directly proportional to the rate of shear strain, is called Newtonian fluid.

$$r = \mu \frac{du}{dy}$$

14. Explain the Non-Newtonian fluid

The fluids which does not obey the Newton's law of viscosity i.e., the shear stress is not directly proportional to the ratio of shear strain, is called non-Newtonian fluid.

15. Define compressibility.

Compressibility is the reciprocal of bulk modulus of elasticity, k which is defined as the ratio of compressive stress to volume strain.

$$k = \frac{\text{Increase of pressure}}{\text{Volume strain}}$$

$$\text{Compressibility } \frac{1}{k} = \frac{\text{Volume of strain}}{\text{Increase of pressure}}$$

16. Define surface tension.

Surface tension is defined as the tensile force acting on the surface of a liquid in Contact with a gas or on the surface between two immiscible liquids such that contact surface behaves like a membrane under tension.

17. Define Capillarity.

Capillary is a phenomenon of rise or fall of liquid surface relative to the adjacent general level of liquid.

18. What is cohesion and adhesion in fluids?

Cohesion is due to the force of attraction between the molecules of the same liquid. Adhesion is due to the force of attraction between the molecules of two different Liquids or between the molecules of the liquid and molecules of the solid boundary surface.

19. State momentum of momentum equation?

It states that the resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum.

20. What is momentum equation

It is based on the law of conservation of momentum or on the momentum principle It states that,the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction.

21. What is Euler's equation of motion

This is the equation of motion in which forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream line.

22. What is venturi meter?

Venturi meter is a device for measuring the rate of fluid flow of a flowing fluid through a pipe. It consisits of three parts.

a. A short converging part b. Throat c.Diverging part.

It is based on the principle of Bernoalli's equation.

23. What is an orifice meter?

Orifice meter is the device used for measuring the rate of flow of a fluid through a pipe. it is a cheaper device as compared to venturi meter. it also works on the priniciple as that of venturi meter. It consists of a flat circular plate which has a circular sharp edged hole called orifice.

24. What is a pitot tube?

Pitot tube is a device for measuring the velocity of a flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of kinetic energy into pressure energy.

. What are the types of fluid flow?

Steady & unsteady fluid flow
Uniform & Non-uniform flow
One dimensional, two-dimensional & three-dimensional flows
Rotational & Irrotational flow

25. State the application of Bernouillie's equation ?

It has the application on the following measuring devices.

1.Orifice meter.
2.Venturimeter.
3.Pitot tube.

PART-B

1.

Calculate the specific weight, mass density, specific gravity and specific volume of oil having a volume of 4.5m³ and weight of 40kN.

Given data:

Volume of oil $= 4.5 m^3$

Weight of oil $= 40 kN = 40 \times 10^3 N$

Solution:

1. Specific weight:

$$w = \frac{\text{Weight of oil}}{\text{Volume of oil}} = \frac{40\times10^3}{4.5} = \mathbf{8.889 \times 10^3 N/m^3}$$ **Ans.**

2. Mass density of oil:

$$\rho = \frac{\text{Specific weight of oil}}{\text{Acceleration due to gravity}} = \frac{w}{g}$$

$$= \frac{8.889\times10^3}{9.81} = \mathbf{906.1 kg/m^3}$$ **Ans.**

3. Specific gravity of oil:

$$S = \frac{\text{Specific weight of oil}}{\text{Specific weight of water}} = \frac{8.889\times10^3}{9.81\times10^3} = \mathbf{0.906}$$ **Ans.**

4. Specific volume of oil:

$$v = \frac{1}{\rho} = \frac{1}{906.1} = \mathbf{1.1 \times 10^{-3} m^3/kg}$$ **Ans.**

2.`

If a liquid has a viscosity of 0.051poise and kinematic viscosity 0.14stokes, calculate its specific gravity.

Given data:

Viscosity, $\mu = 0.051\ poise = 0.0051 Ns/m^2$.

Kinematic viscosity, $\nu = 0.14\ stokes = 0.14 \times 10^{-4}\ m^2/sec$

☺ ***Solution:***

We know that,

$$\nu = \frac{\mu}{\rho}$$

$$\therefore \rho = \frac{\mu}{\gamma} = \frac{0.0051}{0.14 \times 10^{-4}} = 364.28 kg/m^3$$

$$\text{Specific gravity, } S = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{364.28}{1000} = \mathbf{0.364}$$ **Ans.**

$$\nu = \frac{1}{\rho} = \frac{1}{662.59} = \mathbf{1.51 \times 10^{-3} m^3/kg}$$ **Ans.**

Determine the density, specific weight and specific volume of air at 1.1bar and 20℃. Assume the characteristic equation for gases as pV = mRT. Take R = 287J/kgK

Given data:

Pressure, $p = 1.1 bar = 1.1 \times 10^5 N/m^2$

Temperature, $T = 20°C = 20+273 = 293K$

$R = 287 J/kgK$

$pV = mRT$

☺ ***Solution:***

1. Density: $\rho = \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{m}{V}$

From characteristic *gas* equation

$$\rho = \frac{m}{V} = \frac{p}{RT} = \frac{1.1\times10^5}{287\times293} \qquad [\because\ pV = mRT]$$

$$= \mathbf{1.3\ kg/m^3} \qquad \textbf{Ans.}$$

2. Specific weight:

3.

$$w = \rho \times g = 1.3 \times 9.81 = \mathbf{12.75\ N/m^3} \qquad \textbf{Ans.}$$

4.

A soap bubble of 60mm diameter has a gauge pressure $2N/m^2$. Estimate the sur tension of soap bubble.

Given data:

Diameter of soap bubble, $d = 60mm = 0.06m$

Gauge pressure, $p = 2N/m^2$

☺ ***Solution:***

For soap bubble, we know that $p = \frac{8\sigma}{d}$

$$2 = \frac{8\sigma}{0.06}$$

$$\sigma = \mathbf{0.015}N/m$$ **Ans.**

5.

Water is flowing through a tapering pipe having diameters 300mm and 150mm at sections 1 and 2 respectively. The discharge through the pipe is 40lit/s. The section 1 is 10m above datum and section 2 is 6m above datum. Find pressure at section 2, if that at section 1 is 400kN/m²

Given data:

$D_1 = 300mm = 0.3m$

$D_2 = 150mm = 0.15m$

$Q = 40lit/s = 40 \times 10^{-3} m^3/s$

$z_1 = 10m$

$z_2 = 6m$

$p_1 = 400kN/m^2$

☺ ***Solution:***

Velocity of fluid at section 1,

$$v_1 = \frac{Q}{\text{Area}} = \frac{40\times10^{-3}}{\pi/4\,(0.3)^2} = 0.5659m/s$$

Velocity of fluid at section 2,

$$v_2 = \frac{Q}{\text{Area}} = \frac{40\times10^{-3}}{\pi/4\,(0.15)^2} = 2.264m/s$$

Substituting the above values in Bernoulli's equation,

$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2$$

$$\frac{400}{9.81\times1000} + \frac{(0.5659)^2}{2\times9.81} + 10 = \frac{p_2}{9.81\times1000} + \frac{(2.264)^2}{2\times9.81} + 6$$

$$p_2 = \mathbf{37.24kN/m^2}$$

6.

A venturimeter is used for the measurement of discharge of water in a horizontal pipeline. The upstream diameter is 300mm. The throat is 150mm diameter and the pressure difference between inlet and throat is 3m head of water. If the loss of head through converging section of the meter is 1/8 of the throat velocity head, calculate the discharge in the pipeline.

Given data:

$d_1 = 300mm = 0.3m$

$d_2 = 150mm = 0.15m$

$h = 3m$

$h_L = 1/8$ Kinetic head

☺ ***Solution:***

Area at inlet section, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.3)^2 = 0.07m^2$

Area at throat section, $a_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} \times (0.15)^2 = 0.017m^2$

Loss of Kinetic head, $h_L = \frac{1}{8} \times \frac{v_1^2}{2g}$

We know that pressure head,

$$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = 3m$$

Applying Bernoulli's equation

$$\frac{p_1}{w}+\frac{v_1^2}{2g}+z_1 = \frac{p_2}{w}+\frac{v_2^2}{2g}+z_2+h_L$$

$$\left(\frac{p_1}{w}+z_1\right)-\left(\frac{p_2}{w}+z_2\right) = \frac{v_2^2-v_1^2}{2g}+\frac{1}{8}\times\frac{v_1^2}{2g}$$

$$3 = \frac{v_2^2-v_1^2}{2g}+\frac{1}{8}\times\frac{v_1^2}{2g}$$

By continuity equation,

$$Q = a_1v_1 = a_2v_2$$

$$\therefore 0.07 \times v_1 = 0.017v_2$$

$$v_2 = 4v_1$$

$$\therefore\ 3 = \frac{(4v_1)^2-v_1^2}{2\times 9.81}+\frac{v_1^2}{8\times 2\times 9.81}$$

$$3\times 2\times 9.81 = \frac{(16\times v_1^2-v_1^2)}{1}+\frac{v_1^2}{8}$$

$$470.88 = (16\times v_1^2-v_1^2)\,8+v_1^2$$

$$v_1 = 1.97 m/s$$

$\therefore$ Mass flow rate

$$Q = a_1v_1 = 0.07\times 1.97 = 0.138 m^3/s = \mathbf{138\,lit/s}$$

7.

1.8. APPLICATION OF CONTINUITY EQUATION

The continuity equation is governed from the *principle of conservation of mass*. It states that the mass of fluid flowing through the pipe at all cross-section remains constant, if there is no fluid is added or removed from the pipe. Consider two cross-sections of a pipe as shown in Figure 1.37.

8.

1. Continuity equation in differential form

Consider a fluid element of lengths dx, dy and dz in the direction x, y and z. Let u, v and w are the inlet velocities components in X, Y and Z directions respectively.

Rate of mass of fluid entering the face $ABCD$

$$= \rho \times \text{Velocity in } x\text{-direction} \times \text{area of } ABCD = \rho\, u\, dy\, dz$$

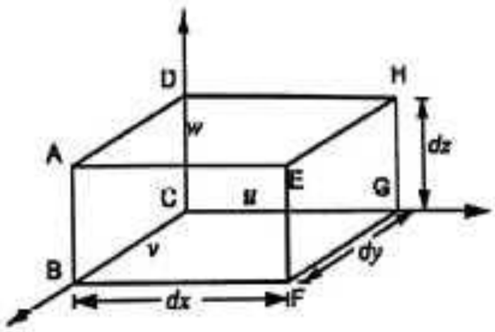

Figure 1.38

Rate of mass of fluid leaving the face $EFGH$

$$= \rho\, u\, dy\, dz + \frac{\partial}{\partial x}\left(\rho u\, dy\, dz\right) dx$$

Gain in mass per unit time due to flow in the x-direction is given by the difference between the fluid entering and leaving.

i.e. Gain of mass in x-direction

$$= \text{Mass through } EFGH - \text{Mass through } ABCD$$

$$= \rho\, u\, dy\, dz + \frac{\partial}{\partial x}\left(\rho u\, dy\, dz\right) dx - \rho\, u\, dy\, dz$$

$$= \frac{\partial}{\partial x}\left(\rho u\, dy\, dz\right) dx$$

$$= \frac{\partial}{\partial x}\left(\rho u\right) \times dx\, dy\, dz \qquad (\because dy\, dz \text{ is constant})$$

Similarly, the gain in fluid mass per unit time due to flow in Y and Z direction

$$= \frac{\partial}{\partial x}\left(\rho v\right) dx\, dy\, dz \quad (\text{in } Y\text{ - direction})$$

$$= \frac{\partial}{\partial x}(\rho w)\, dx\, dy\, dz \quad \text{(in Z - direction)}$$

$$\therefore \text{ Net gain} = \left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\, dy\, dz$$

According to the principle of conservation of mass, there is no accumulation of mass and hence the above quantity must be zero.

$$\text{i.e. } \left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right] dx\, dy\, dz = 0$$

$$\boxed{\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0}$$

This equation is the *general equation of continuity* in three-dimensions which is applicable to any type of flow and for any fluid whether compressible or incompressible.

For *incompressible fluids,* $\rho = 0$, then the above equation reduces to

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

For *two-dimensional flow,* the component $w = 0$ and hence, the continuity equation becomes

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$$

2. Continuity equation in polar co-ordinates

The equation of continuity in polar co-ordinates, for incompressible fluids may be written as follows

$$\frac{v_r}{r} + \frac{\partial v_r}{\partial r} + \frac{\partial v_\theta}{r\partial\theta} = 0$$

where, $v_r \Rightarrow$ Velocity component in radial direction.

$v_\theta \Rightarrow$ Velocity component in tangential direction.

1.9.1. Euler's equation

Consider a steady flow of an ideal fluid along a streamline with a small element of the flowing fluid LM of cross section dA and length ds as shown in Figure 1.39.

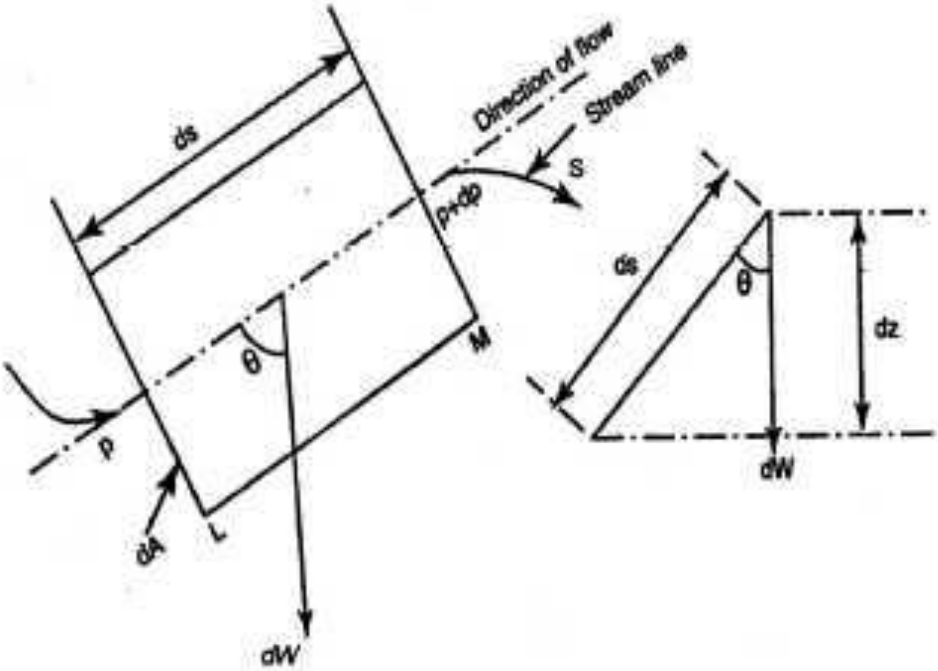

Figure 1.39

Let,

p = Pressure on the element at A.

$p + dp$ = Pressure on the element at M and

v = Velocity of the fluid element

We know that the net force acting on the fluid element in the direction of flow

$$= p.\, dA - (p + dp)\, dA = -\, dp\, dA$$

We also know that the weight of the fluid element

$$dw = \rho g \,.\, dA \,.\, ds$$

From the geometry of the Figure 1.39, we find that the component of weight of the fluid element in the direction of flow.

$$= -\rho g \,.\, dA \,.\, ds \,.\, \cos\theta = -\rho \,.\, g \,.\, dA \,.\, ds \,.\left(\frac{dz}{dS}\right) \qquad \left(\because \cos\theta = \frac{dz}{ds}\right)$$

$$= -\rho \,.\, g.\, dA \,.\, dz$$

The resultant force on the fluid element in the direction of S

$$= -dp.\, dA - \rho \,.\, g.\, dA \,.\, dz$$

$\therefore$ Mass of the fluid element = $\rho \,.\, dA \,.\, ds$

The acceleration of the fluid element

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v.\frac{dv}{ds}$$

According to the Newton's second law of motion,

Force = Mass × Acceleration

$$(-dp.dA) - (\rho g.dA.dz) = \rho\, dA\, ds \times v \times \frac{dv}{ds}$$

Dividing both sides by $\rho \,.\, dA$, then the above equation becomes

$$-\frac{dp}{\rho} - g\, dz = v \times dv$$

$$\boxed{\frac{dp}{\rho} + v\, dv + g\,.dz = 0}$$

The above equation is known as ***Euler's equation*** of motion. It is in the form of differential equation.

1.9.2. Bernoulli's Equation from Euler's Equation

Integrating the above equation,

$$\frac{1}{\rho}\int dp + \int g.dz + \int v.dv = \text{Constant}$$

$$\frac{p}{\rho} + g\,z + \frac{v^2}{2} = \text{Constant}$$

Dividing by g, $\dfrac{p}{\rho g} + \dfrac{v^2}{2g} + z = \text{Constant}$

$$\frac{p}{w} + \frac{v^2}{2g} + z = \text{Constant}$$

$$\frac{p_1}{w} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{w} + \frac{v_2^2}{2g} + z_2$$

It proves the *Bernoulli's equation.*

1.9.3. Bernoulli's Equation

Bernoulli's Equation relates velocity, pressure and elevation changes of a fluid in motion. It may be stated as follows "*In an ideal, incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential energy is constant along a streamline*".

Mathematically,

$$\frac{p}{w} + \frac{v^2}{2g} + z = \text{Constant.}$$

where $\frac{p}{w} \Rightarrow$ Pressure energy.

$\frac{v^2}{2g} \Rightarrow$ Kinetic energy

$z \Rightarrow$ Datum energy

Proof:

Consider steady the flow of incompressible liquid through a non-Uniform pipe lying entirely in the *x-y* plane.

The following assumptions are made in the derivation of Bernoulli's Equation.

- The liquid is ideal and incompressible.
- The flow is steady and continuous.
- The velocity is uniform over the cross section and is equal to mean velocity.
- The only forces acting on the fluids are the gravity forces and the pressure forces.

$$\frac{p_1}{w}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{w}+\frac{v_2^2}{2g}+z_2$$

It proves the *Bernoulli's equation.*

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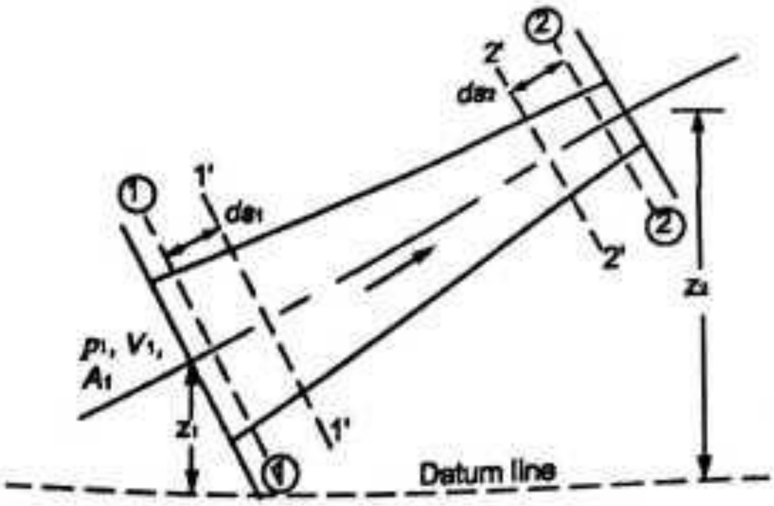


Figure 1.40

Let us consider two sections 1-1 and 2-2 in an uniform varying pipe as shown in Figure 1.40.

Let, $p_1 \Rightarrow$ pressure at 1-1.

$v_1 \Rightarrow$ Velocity at 1-1.

$z_1 \Rightarrow$ Height of 1-1 above the datum.

$A_1 \Rightarrow$ Area of pipe at 1-1and

p_2, v_2, z_2 and A_2 are corresponding values at 2-2.

The mass of the fluid in the region 1-1 and 2-2 shifts to new position 1′-1′ and 2′-2 during an infinitely small interval of time. The movement of liquid between 1-1 and 2 2 is equivalent to the movement of the liquid between 1-1 and 1′-1′ to 2-2 and 2′-2 the remaining liquid between 1′-1′ and 2-2 being unaffected. Invoking the principle o conservation of mass, the following continuity equation applies.

Fluid mass within the region 1-1 and 1′-1′ = Fluid mass within the region 2-2 and 2′-2

$$m = \rho A_1 ds_1 = \rho A_2 ds_2$$

Work done by the pressure at 1-1 in moving liquid to 1′-1′

$$= \text{Force} \times \text{distance} = p_1 A_1 ds_1$$

Similarly Work done by the pressure at 2-2 in moving liquid to 2'-2'

$= -p_2 A_2 ds_2$ ($\because$ – ve sign indicates that direction p_2 opposite to that of p_1)

$\therefore$ Total work done by the pressure

$= p_1 A_1 ds_1 - p_2 A_2 ds_2 = A_1 ds_1 (p_1 - p_2)$ ($\because A_1 ds_1 = A_2 ds_2$)

$= \dfrac{m}{\rho}(p_1 - p_2)$

Loss of potential energy = $mg(z - z_2)$

Gain of Kinetic energy = $\dfrac{m}{2}(v_2^2 - v_1^2)$

From the principle of conservation of energy,

Loss of potential energy + work done by the pressure = Gain in Kinetic energy

$$mg(z_1 - z_2) + \frac{m}{\rho}(p_1 - p_2) = \frac{m}{2}(v_2^2 - v_1^2)$$

Dividing throughout by mg and using the relation $w = \rho g$

$$(z_1 - z_2) + \frac{(p_1 - p_2)}{\rho g} = \frac{(v_2^2 - v_1^2)}{2g}$$

Rearranging the above equation,

$$\frac{v_1^2}{2g} + \frac{p_1}{w} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{w} + z_2 \quad \text{or}$$

$$\frac{p}{w} + \frac{v^2}{2g} + z = \text{Constant.}$$

It is *Bernoulli's equation.*

10.

Water flows through a pipe AB of diameter 50mm, which is in series with pipe BC of diameter 75mm in which the velocity is 2m/s. At C the pipe forks and one branch CD is of unknown diameter such that the velocity is 1.5m/s. The other branch CE is of diameter 25mm and condition are such that the discharge in the pipe BC divides so that the discharge in the pipe CD is equal to two times of discharge in CE. Calculate (i) the discharge in pipe AB and CD. (ii) velocity in pipe AB and CE. (iii) diameter of pipe CD.

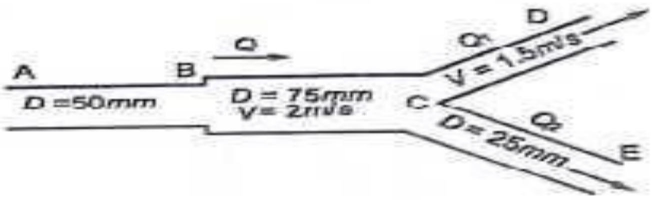


Figure 1.56

Pipe AB:

Cross sectional area of pipe AB

$$A_{AB} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.05)^2 = 1.9635 \times 10^{-3} m^2$$

Pipe BC:

Cross sectional area of pipe BC

$$A_{BC} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.075)^2 = 4.4179 \times 10^{-3} m^2$$

Pipe CE:

Cross sectional area of pipe CE

$$A_{CE} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.025)^2 = 4.909 \times 10^{-4} m^2$$

Mass flow rate in pipe AB

$$\therefore A_{AB} \times V_{AB} = A_{BC} \times V_{BC} = 4.4179 \times 10^{-3} \times 2$$

$$Q_{BC} = Q_{AB} = \mathbf{8.8358 \times 10^{-3} m^3/s}$$

Velocity in pipe AB

$$A_{AB} \times V_{AB} = 8.8358 \times 10^{-3}$$

$$\therefore V_{AB} = \frac{8.8358 \times 10^{-3}}{1.9635 \times 10^{-3}} = \mathbf{4.5 m/s}$$

Mass flow rate in BC = Mass flow rate in CD + Mass flow rate in CE

$$Q_{BC} = Q_{CD} + Q_{CE}$$

$$Q_{BC} = 2Q_{CE} + Q_{CE}$$

$$8.8358 \times 10^{-3} = 3Q_{CE}$$

$$\therefore Q_{CE} = 2.9453 \times 10^{-3} m^3/s$$

Velocity in Pipe CE

$$Q_{CE} = A_{CE} \times V_{CE}$$

$$V_{CE} = \frac{Q_{CE}}{A_{CE}} = \frac{2.9453\times10^{-3}}{4.909\times10^{-4}} = \mathbf{6 m/s}$$ **Ans.**

Diameter of Pipe CD

$$Q_{CD} = 2 \times Q_{CE} = 2 \times 2.9453 \times 10^{-3} = \mathbf{5.8906 \times 10^{-3} m^3/s}$$ **Ans.**

$$Q_{CD} = A_{CD} \times V_{CD}$$

$$A_{CD} = \frac{Q_{CD}}{V_{CD}} = \frac{5.8906\times10^{-3}}{1.5} = 3.9271 \times 10^{-3} m^2$$

$$\therefore A_{CD} = \frac{\pi}{4} d^2 = 3.9271 \times 10^{-3}$$

$$d = \sqrt{\frac{3.9271\times10^{-3}\times4}{\pi}} = \mathbf{70.7 mm}$$ **Ans.**

11.

An oil of specific gravity 0.85 is flowing through an inclined venturimeter fitted to a 250mm diameter pipe at the rate of 110lit/s. The venturimeter is inclined at 60° to the vertical and its 120mm diameter throat is 1m from the entrance along its length. The pressure gauges inserted at entrance and throat show pressures of 0.125N/mm² and 0.08N/mm² respectively. Calculate the discharge coefficient of venturimeter. If instead of pressure gages the entrance and throat of the venturimeter are connected to the two limbs of a U-tube mercury manometer, determine its reading in m of mercury column.

Given data:

Specific gravity of oil, $S = 0.85$

Diameter of pipe, $d_1 = 250mm = 0.25m$

Rate of flow, $Q = 110lit/s = 0.11m^3/s$

Inclination to vertical, $\theta = 60°$

Throat diameter, $d_2 = 120mm = 0.12m$

Distance of throat from entrance $= 1m$

Pressure at the entrance, $p_1 = 0.125N/mm^2 = 0.125 \times 10^6 N/m^2$

Pressure at the throat, $p_2 = 0.08N/mm^2 = 0.08 \times 10^6 N/m^2$

☺ ***Solution:***

$$\text{Area of entrance, } a_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4} \times (0.25)^2 = 0.049m^2$$

$$\text{Area at throat section, } a_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4} \times (0.12)^2 = 0.0113m^2$$

$$\text{Pressure head at entrance, } \frac{p_1}{w} = \frac{0.125 \times 10^6}{9810 \times 0.85} = 14.99m \text{ of oil}$$

$$\text{Pressure head at throat, } \frac{p_2}{w} = \frac{0.08 \times 10^6}{9810 \times 0.85} = 9.59m \text{ of oil}$$

$$z_1 = 0$$

$$z_2 = 1 \times \text{Cos } 60 = 0.5m$$

We know that,

$$h = \left(\frac{p_1}{w} + z_1\right) - \left(\frac{p_2}{w} + z_2\right) = (14.99 + 0) - (9.59 + 0.5) = 4.9m$$

The discharge through the venturimeter is given by

$$Q = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$0.11 = C_d \times \frac{0.049 \times 0.0113}{\sqrt{(0.049)^2 - (0.0113)^2}} \times \sqrt{2 \times 9.81 \times 4.9}$$

$$0.11 = C_d \times 0.11386$$

$$C_d = \mathbf{0.966}$$

Ans.

If a U-tube manometer is connected then

$$h = x\left(\frac{S_m}{S} - 1\right)$$

$$4.9 = x\left(\frac{13.6}{0.85} - 1\right)$$

$$x = \mathbf{0.326m}$$

12.

A bend in pipeline converging water gradually reduces from 700mm to 400mm diameter and deflects the flow through an angle of 45°. Find the magnitude and direction of force exerted on the bend, if the velocity of flow at 700mm section is 8m/s and pressure is 350kN/m².

Given data:

Inlet diameter, $D_1 = 700mm = 0.7m$

Outlet diameter, $D_2 = 400mm = 0.4m$

Angle of bend, $\theta = 45°$

Velocity of flow at inlet, $v_1 = 8m/s$

Pressure at inlet, $p_1 = 350kN/m^2$

Solution:

Area at inlet, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (0.7)^2 = 0.3848m^2$

Area at outlet, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (0.4)^2 = 0.12566m^2$

From continuity equation,

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{0.3848 \times 8}{0.12566} = 24.497m/s$$

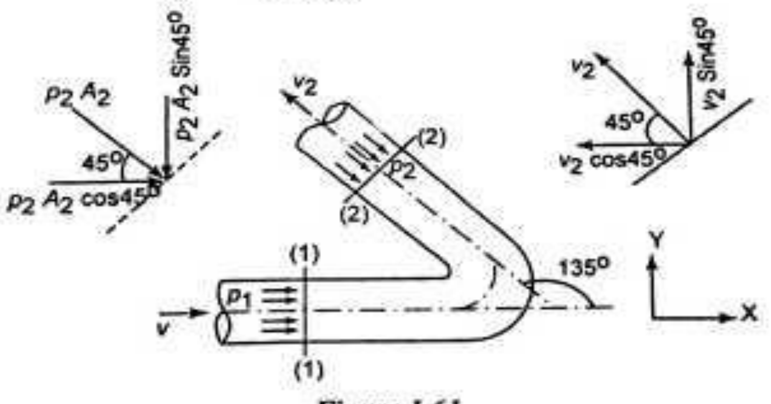


Figure 1.61

Applying Bernoulli's equation at sections (1) and (2),

$$\frac{p_1}{w} + \frac{v_1^2}{2g} = \frac{p_2}{w} + \frac{v_2^2}{2g} \qquad (\because z_1 = z_2)$$

$$\frac{350}{9.81} + \frac{8^2}{2 \times 9.81} = \frac{p_2}{9.81} + \frac{(24.497)^2}{2 \times 9.81}$$

$$p_2 = 81.948kN/m^2$$

Discharge, $Q = A_1 v_1 = 0.3848 \times 8 = 3.0784 m^3/s$

Force along x direction is given in equation (3.1)

$$F_x = \rho Q (v_1 - v_2 \cos\theta) - p_2 A_2 \cos\theta + p_1 A_1$$

$$= [1000 \times 3.0784 (8 - 24.497 \times \cos 45)] - \begin{bmatrix} 81.948 \times 10^3 \times \\ 0.12566 \times \cos 45 \end{bmatrix} + 350 \times 10^3 \times 0.3848$$

$$F_x = 98701.678 N$$

Force along y-direction (from equation 3.2)

$$F_y = \rho Q v_2 \sin\theta + p_2 A_2 \sin\theta$$

$$= (1000 \times 3.0784 \times 24.497 \times \sin 45) + (81.948 \times 10^3 \times 0.12566 \times \sin 45)$$

$$= 60605.52 N$$

Resultant force, $F_R = \sqrt{F_x^2 + F_y^2}$

$$= \sqrt{(98701.678)^2 + (60605.52)^2} = \mathbf{115823.36 N} \quad \textbf{Ans.}$$

The direction of resultant force with x axis is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{60605.52}{98701.678} = 0.614$$

$$\therefore \theta = \tan^{-1}(0.614) = \mathbf{31.55°}$$

Ans.

UNIT II

FLOW THROUGH CIRCULAR CONDUITS

PART – A

1. Define viscosity (u).

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.Viscosity is also defined as the shear stress required to produce unit rate of shear strain.

2. Define kinematic viscosity.

Kniematic viscosity is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by μ.

3. What is minor energy loss in pipes?

The loss of head or energy due to friction in a pipe is known as major loss while loss of energy due to change of velocity of fluid in magnitude or direction is called minor loss of energy. These include,

a. Loss of head due to sudden enlargement.
b. Loss of head due to sudden contraction.
c. Loss of head at entrance to a pipe.
d. Loss of head at exit of a pipe.
e. Loss of head due to an obstruction in a pipe.
f. Loss of head due to bend in a pipe.
g. Loss of head in various pipe fittings.

4. What is total energy line?

Total energy line is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing sum of the pressure head and kinetic head from the centre of the pipe.

5. What is hydraulic gradient line?

Hydraulic gradient line gives the sum of (p/w+z) with reference to datum line. Hence hydraulic gradient line is obtained by subtracting v2 / 2g from total energy line.

6. What is meant by pipes in series?

When pipes of different lengths and different diameters are connected end to end, pipes are called in series or compound pipe. The rate of flow through each pipe connected in series is same.

7. What is meant by pipes in parallel?

When the pipes are connected in parallel, the loss of head in each pipe is same. The rate of flow in main pipe is equal to the sum of rate of flow in each pipe, connected in parallel.

8. What is boundary layer and boundary layer theory?

When a solid body immersed in the flowing fluid, the variation of velocity from zero to free stream velocity in the direction normal to boundary takes place in a narrow region in the vicinity of solid boundary. This narrow region of fluid is called boundary layer. The theory dealing with boundary layer flow is called boundary layer theory.

9. What is turbulent boundary layer?

If the length of the plate is more then the distance x, the thickness of boundary layer will go on increasing in the downstream direction. Then laminar boundary becomes unstable and motion of fluid within it, is disturbed and irregular which leads to a transition from laminar to turbulent boundary layer.

10. What is boundary layer thickness?

Boundary layer thickness (S) is defined as the distance from boundary of the solid body measured in y-direction to the point where the velocity of fluid is approximately equal to 0.99 times the free steam (v) velocity of fluid.

11. Define displacement thickness

Displacement thickness (S*) is defined as the distances, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction inflow rate on account of boundary layer formation.

12. What is momentum thickness?

Momentum thickness (0) is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in momentum of flowing fluid on account of boundary layer formation.

13.Mention the general characteristics of laminar flow.

- There is a shear stress between fluid layers
- 'No slip' at the boundary
- The flow is rotational
- There is a continuous dissipation of energy due to viscous shear

14. What is Hagen poiseuille's formula ?

P_1-P_2 / ρg = h $_f$ = 32 μUL / _gD^2

The expression is known as Hagen poiseuille formula .

Where P_1-P_2 / _g = Loss of pressure head U = Average velocity

μ = Coefficient of viscosity D = Diameter of pipe

L = Length of pipe

15.What are the factors influencing the frictional loss in pipe flow ?

Frictional resistance for the turbulent flow is

i. Proportional to v_n where v varies from 1.5 to 2.0 . ii. Proportional to the density of fluid .

iii. Proportional to the area of surface in contact . iv. Independent of pressure .

v. Depend on the nature of the surface in contact .

16. What is the expression for head loss due to friction in Darcy formula ?

$h_f = 4fLV^2 / 2gD$

Where f = Coefficient of friction in pipe L = Length of the pipe

D = Diameter of pipe V = velocity of the fluid

17. What do you understand by the terms

a) major energy losses , b) minor energy losses

Major energy losses : -

This loss due to friction and it is calculated by Darcy weis bach formula and chezy's formula .

Minor energy losses :- This is due to

i. Sudden expansion in pipe .ii. Sudden contraction in pipe.

iii. Bend in pipe .iv. Due to obstruction in pipe .

18. Give an expression for loss of head due to sudden enlargement of the pipe :

$he = (V1-V2)^2 /2g$

Wherehe = Loss of head due to sudden enlargement of pipe .

V1 = Velocity of flow at section 1-1

V2 = Velocity of flow at section 2-2

19.Give an expression for loss of head due to sudden contraction:

$hc = 0.5 V^2/2g$

Where hc = Loss of head due to sudden contraction .

V = Velocity at outlet of pipe.

20. Give an expression for loss of head at the entrance of the pipe

hi =0.5V2/2g

where hi = Loss of head at entrance of pipe .

V = Velocity of liquid at inlet and outlet of the pipe .

21. What is sypon ? Where it is used: _

Sypon is along bend pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level .

Uses of sypon : -

1. To carry water from one reservoir to another reservoir separated by a hill ridge .
2. To empty a channel not provided with any outlet sluice.

PART-B

1.

A horizontal pipe of 250mm diameter and 60m long is connected to a water tank at one end and discharges freely to atmosphere through the other end. If height of the water in the tank is 4.5m above the centre of the pipe, calculate the rate of flow of water. Consider all losses and take f = 0.008. Also draw the Hydraulic grade line (H. G. L) and total energy line (T.E.L).

Given data:

Diameter of pipe, $D = 250\ mm = 0.25m$

Length of pipe, $L = 60m$

Height of water, $H = 4.5m$

Co-efficient of friction, $f = 0.008$

☺ ***Solution:***

Head loss at the entrance of the pipe, $h_i = \dfrac{0.5V^2}{2g}$

Head loss due to friction in the pipe, $h_f = \dfrac{4fLV^2}{2gD}$

Head loss at the exit from a pipe, $h_o = \dfrac{V^2}{2g}$

Applying Bernoulli's equation at the top of the water surface in the tank and at t outlet of the pipe,

$$\frac{p_1}{w}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{w}+\frac{V_2^2}{2g}+z_2+\text{All losses}$$

$$0+0+4.5=0+\frac{V_2^2}{2g}+0+\frac{0.5V^2}{2g}+\frac{4fLV^2}{2gD}+\frac{V^2}{2g}$$

But the velocity in the pipe $V=V_2$

$$4.5=\frac{V^2}{2g}\left[1+0.5+\frac{4fL}{D}+1\right]$$

$$=\frac{V^2}{2g}\left[1+0.5+\frac{4\times0.008\times60}{0.25}+1\right]$$

$$4.5=\frac{V^2}{2g}\times(10.18)$$

$$V=\sqrt{\frac{4.5\times2\times9.81}{10.18}}=2.945m/s$$

Rate of flow, $Q=A\times V$

$$=\frac{\pi}{4}\times(0.25)^2\times2.945=\mathbf{0.1445m^3/s}$$ **Ans.**

Hydraulic Gradient Line (H. G. L.) gives the sum of $\left(\frac{p}{w}+z\right)$ with reference to the datum line. Hence, H. G. L. is obtained by subtracting $\frac{V^2}{2g}$ from total energy available at that point.

Head loss at the entrance of the pipe, $h_i=\frac{0.5\times(2.945)^2}{2\times9.81}=0.221m$

Head loss due to friction, $h_f=\frac{4fLV^2}{2gD}=\frac{4\times0.008\times60\times(2.945)^2}{2\times9.81\times0.25}=3.3949m$

Head loss at exit of the pipe, $h_o=\frac{V^2}{2g}=\frac{(2.945)^2}{2\times9.81}=0.442m$

Total energy available at the entrance of the pipe

$$= h - h_i = 4.5 - 0.221 = 4.279m$$

The piezometric head $\left(\frac{p}{w}+z\right)$ at the entrance= Total energy at entrance $-\frac{V^2}{2g}$ at

entrance of the pipe $= 4.2749 - \frac{(2.945)^2}{2\times 9.81} = 3.833m$

Similarly, total energy at exit of the pipe,

$$= h - (h_i + h_f + h_o)$$

$$= 4.5 - (0.221 + 3.3949 + 0.442)\ 0.442m$$

The piezometer head $\left(\frac{p}{w}+z\right)$ available at exit of the pipe

$$= 0.442 - \frac{V^2}{2g} = 0.442 - \frac{(2.945)^2}{2\times 9.81} = 0m$$

Total energy line (T.E.L):

1. Point A lies on the free surface of water since total energy at A = $\frac{p}{w} + \frac{V^2}{2g} + z = 0 + 0 + 4.5 = 4.5m$.

2. A point B is noted at a distance AB = h_i = 0.221 m because the total energy at entrance of the pipe B = Total energy at A – h_i = 4.5 – 0.221 = 4.279 m.

3. Total energy available at the exit of the pipe. i.e., at C is already found out as 0.442 m. Therefore, a point C is placed at a distance 0.442 m from the centre line as shown in Figure 2.3.

4. A, B and C are joined by straight lines. Then ABC represents the total energy line.

Hydraulic gradient line (H.G.L):

HGL gives the piezomteric head i.e., (sum of $\frac{p}{w}+z$) with refernce to the datum line.

1. Piezometric head at the entrance of the pipe is already found as 3.836 m. A point D is placed at a distance of 3.836 m from the datum.

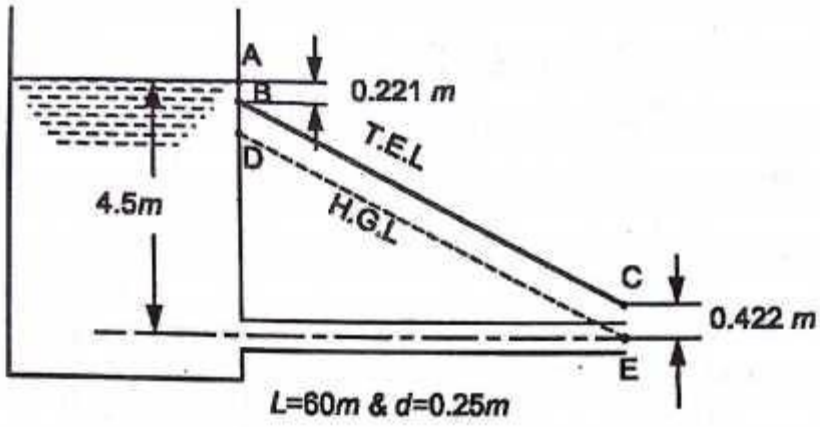


Figure 2.3

2. Piezometric head at the exit of the pipe is 0m. Therefore, a point E is placed on the datum line as shown in Figure 2.3.

3. Points D and E are joined by a straight line. The line DE represents the HGL.

3.

A horizontal pipeline 50m is connected to a water tank at one end and discharges freely to atmosphere through the other end. For the first 30m lengths from tank, the diameter of pipe is 15cm and for rest it is 30cm in diameter. The water level in the tank is 8m above the centre of the pipe. Take $f = 0.01$. By considering all losses, determine the discharge through the pipe. Also draw the hydraulic gradient line and total energy line.

Given data:

Total length of pipe

$L = 50m$

$L_1 = 30m$

$L_2 = 20m$

$D_1 = 15cm = 0.15m$

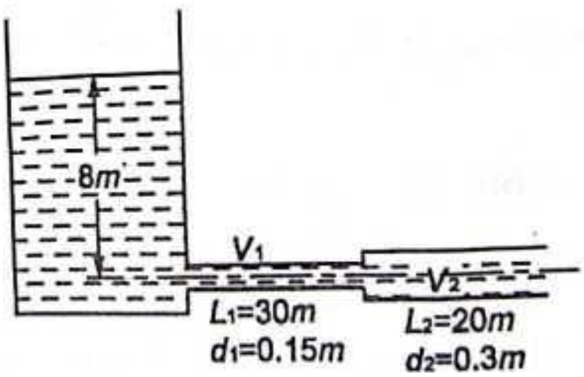
$D_2 = 30cm = 0.3m$

$f = 0.01$

☺ ***Solution:***

Losses in the pipeline:

Head loss at the entrance of the pipe, $h_i = \dfrac{0.5V_1^2}{2g}$



Head loss due to friction in the pipe (1), $h_{f_1} = \dfrac{4 f L_1 V_1^2}{2gD_1}$

Head loss due to sudden enlargement, $h_e = \dfrac{(V_1 - V_2)^2}{2g}$

Head loss due to friction in pipe (2), $h_{f_2} = \dfrac{4 f L_2 V_2^2}{2gD_2}$

Head loss at the exit from a pipe, $h_o = \dfrac{V^2}{2g}$

From continuity equation, $A_1 V_1 = A_2 V_2$

$$V_1 = \frac{A_2 V_2}{A_1} = \frac{\pi_4 \times D_2^2 \times V_2}{\pi_4 \times D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.3}{0.15}\right)^2 \times V_2$$

$$V_1 = 4V_2$$

$$V_2 = 0.25V_1$$

Substituting the value of V_1 in different head losses,

$$h_i = \frac{0.5V_1^2}{2g} = \frac{0.5 \times (4V_2)^2}{2g} = \frac{8V_2^2}{2g}$$

$$h_{f_1} = \frac{4 f L_1 V_1^2}{2gD_1} = \frac{4 \times 0.01 \times 30 \times (4V_2)^2}{2 \times 9.81 \times 0.15} = 6.52V_2^2$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2 \times 9.81} = 0.458\ V_2^2$$

$$h_{f_2} = \frac{4 f L_2 V_2^2}{2gD_2} = \frac{4 \times 0.01 \times 20 \times (4V_2)^2}{2 \times 9.81 \times 0.3} = 2.1746V_2^2$$

$$h_o = \frac{V_2^2}{2g} = 0.051V_2^2$$

Applying Bernoulli's theorem to the force surface of water in the tank and outlet of pipe,

$$\frac{p_1}{w}+\frac{V_1^2}{2g}+Z_1=\frac{p_2}{w}+\frac{V_2^2}{2g}+Z_2+\text{All losses}$$

$$0+0+8=0+\frac{V_2^2}{2g}+0+h_i+h_{f_1}+h_e+h_{f_2}+h_o$$

$$8=\frac{V_2^2}{2g}+\frac{8V_2^2}{2g}+6.52V_2^2+0.458V_2^2+2.1746V_2^2+0.051V_2^2$$

$$8=9.66V_2^2$$

$$V_2=\sqrt{\frac{8}{9.66}}=0.91m/s$$

Rate of flow, $Q=A_2V_2=\frac{\pi}{4}\times D_2^2\times V_2$

$$=\frac{\pi}{4}\times(0.3)^2\times0.91=\mathbf{0.0643m^3/s}\qquad\textbf{Ans.}$$

Total energy line and Hydraulic gradient line (H.G.L):

Total energy line gives the total energy $\left(\frac{p}{w}+\frac{V^2}{2g}+z\right)$ with reference to the datum line. H.G.L gives the sum of $\left(\frac{p}{w}+z\right)$ with reference to the datum line. Hence hydraulic gradient line is obtained by subtracting ($V^2/2g$) from total energy at that point.

- A point A is taken on the water surface which is $8m$ above from centre line of the pipe i.e. $h=8m$
- Total head available at the entrance of the pipe,

$$=h-h_i=8-\frac{8V_2^2}{2g}=8-\frac{8\times(0.91)^2}{2\times9.81}=7.662m$$

$\therefore$ Piezometric head $\left(\frac{p}{w}+z\right)$ at the entrance

$$=7.662-\frac{V_1^2}{2g}=7.662-\frac{(4V_2)^2}{2\times9.81}=7.662-\frac{(4\times0.91)^2}{2\times9.81}$$

Piezometric head at the entrance = $6.986m$

Total energy available just before enlargement of the pipe,

$$= \text{Total energy available at the entrance} - h_{f_1}$$

$$= 7.662 - 6.52 V_2^2 = 7.662 - 6.52 \times (0.91)^2 = 2.263\ m$$

Total head available at the enlargement of pipe = Total energy at the entrance of the pipe $- (h_{f_1} + h_e)$

$$= 7.662 - \left(6.52 V_2^2 + 0.4857 V_2^2\right)$$

$$= 7.662 - \left[6.52 \times (0.91)^2 + 0.4857 \times (0.91)^2\right] = 1.86 m$$

$\therefore$ Piezometric head $\left(\frac{p}{w} + z\right)$ at the entrance,

$$= 1.86 - \frac{V_1^2}{2g} = 1.86 - \frac{(4 \times 0.91)^2}{2 \times 9.81} = 1.86 - 1.1856 = 0.674 m$$

Total energy available at the exit of the pipe = Total energy available at the enlargement $- h_{f_2}$

$$= 1.86 - (2.175 V_2^2) = 1.86 - (2.175 \times (0.91)^2) = 0.059\ m$$

Total energy line (T.E.L):

1. A point A is noted on the free surface of water.
2. Total energy available at the entrance of the pipe is 7.662*m*. So, the point B is noted at a distance 7.662 *m* from the datum.
3. Total energy available just before the enlargement of the pipe is 2.263 *m*. So, the point C is noted at 2.263 *m* from the datum.
4. Total energy available at the enlargement of the pipe is 1.86 *m*. The point D is noted at 1.86 *m* from the datum.
5. Total energy available at the exit of the pipe is 0.059*m*. The point E is noted at 0.059 *m* from the centerline of the pipe.

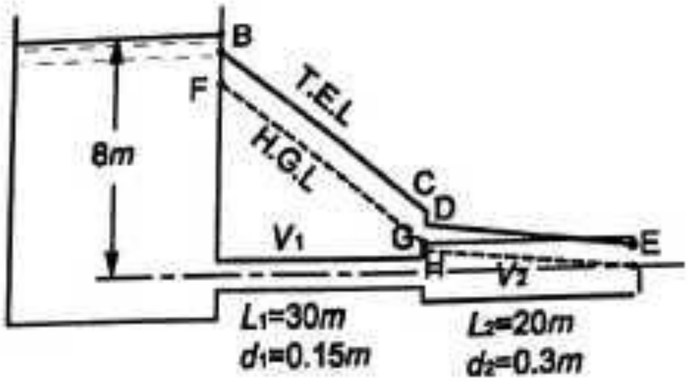


Figure 2.7

Hydraulic gradient line (H.G.L):

HGL gives the piezomteric head i.e., (sum of $\frac{p}{w}+z$) with reference to the datum line.

1. The point F is noted at a distance of 6.986 *m* from the datum at the entrance.
2. A line FG is drawn parallel to the line BC.
3. Generally, piezometric head available at the exit of the pipe is zero due to atmospheric pressure at the exit. The point I is noted on the centre line of the pipe at exit.
4. From I, a line IH is drawn parallel to DE.
5. Point Hand I are joined by a straight line. Then the line FGHI represents the HGL.

4.

An oil of viscosity 1.5N-s/m² flows between two parallel fixed plates which are kept at a distance of 60mm apart. The maximum velocity of oil is 2m/s. Calculate:

1. *The discharge per m length*
2. ***The shear stress at the plates***
3. ***The pressure difference between two points of 10m apart along the direction of flow.***
4. ***The velocity gradient at the plates.***
5. ***The velocity at 18mm from the plate.***

Given data:

Viscosity of oil, $\mu = 1.5 N\text{-}s/m^2$

Distance between plates, $b = 60mm = 0.06m$

Maximum velocity, $U_{max} = 2m/s$

☺ ***Solution:***

(i) Discharge:

$$\text{Average velocity, } U_{ave} = \frac{2}{3} U_{max} = \frac{2}{3} \times 2 = 1.33 m/s$$

$$\text{Discharge, } Q = U_{ave} \times \text{area}$$

$$= 1.33 \times 0.06 \times 1 = \mathbf{0.0798\ m^3/s}$$ **Ans.**

(ii) Shear stress at the plates:

We know that, maximum velocity,

$$U_{max} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right) b^2$$

$$2 = -\frac{1}{8 \times 1.5}\left(\frac{\partial p}{\partial x}\right)(0.06)^2$$

$$\frac{\partial p}{\partial x} = -6666.7\ N/m^2$$

The shear stress is maximum at the plates

$$\tau_{max} = -\frac{1}{2}\left(\frac{\partial p}{\partial x}\right) b$$

$$= -\frac{1}{2} \times (-6666.7) \times 0.06 = \mathbf{200 N/m^2}$$ **Ans.**

(iii) ***Pressure different between two points of 10m parat***

We know that,

$$\frac{\partial p}{\partial x} = -6666.7$$

$$\partial p = -6666.7\ \partial x$$

Integrating with respect to x, we get

$$\int_{p_1}^{p_2} \partial p = \int_{x_1}^{x_2} -6666.7\ \partial x$$

$$p_1 - p_2 = 6666.7\,(x_2 - x_1)$$

$$= 6666.7 \times 10 = \mathbf{66667 N/m^2} \text{ or } = \mathbf{66.67 kN/m^2}$$ **Ans.**

(iv) ***Velocity gradient at the plates:***

We know that $\tau_{max} = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$

$$\left(\frac{\partial u}{\partial y} \right)_{y=0} = \frac{\tau_{max}}{\mu} = \frac{200}{1.5} = \mathbf{133.33 s\text{-}1}$$ **Ans.**

(v) ***Velocity at 18mm from the plate***

$$u = -\frac{1}{2\mu} \frac{\partial p}{\partial x} \left(by - y^2 \right) \qquad (\therefore y = 18mm = 0.018m)$$

$$= -\frac{1}{2 \times 1.5} \times (-6666.7) \times \left[(0.06 \times 0.018) - (0.018)^2 \right]$$

$$= \mathbf{1.68 m/s}$$ **Ans.**

5.

Lubricating oil of specific gravity 0.84 and dynamic viscosity 0.137N-s/m² is pumped at a rate of 0.024m³/s through a 0.17m diameter 400m long horizontal pipe. Calculate the pressure drop, average shear stress at the wall of the pipe and the power required to maintain flow.

Given data:

Specific gravity of oil, $S = 0.84$

Dynamic viscosity of oil, $\mu = 0.137 N\text{-}s/m^2$

Rate of flow, $Q = 0.024 m^3/s$

Diameter of pipe, $D = 0.17 m$

Length of pipe, $L = 400 m$

☺ ***Solution:***

From Hagen-Poiseuille equation,

$$\text{Pressure drop, } (p_1 - p_2) = \frac{128 \mu Q L}{\pi D^4}$$

$$= \frac{128 \times 0.137 \times 0.024 \times 400}{\pi (0.17)^4}$$

$$= \mathbf{64158.79 N/m^2}$$

Ans.

$$\text{Shear stress at the pipe wall, } \tau_{max} = \left(-\frac{dp}{dx}\right) \cdot \frac{R}{2} = \frac{p_1 - p_2}{L} \times \frac{R}{2}$$

$$= \frac{64158.79}{400} \times \frac{0.17}{2 \times 2}$$

$$= \mathbf{6.817 N/m^2}$$

Ans.

6.

The velocity distribution in laminar boundary layer is given by

$$\frac{u}{U}=3\left(\frac{y}{\delta}\right)-2\left(\frac{y}{\delta}\right)^2$$

Where, u ***= Velocity at distance y from the boundary***

U ***= Velocity at a distance*** δ***, the thickness of the boundary layer.***

Calculate:

(i) The ratio of displacement thickness to boundary layer thickness $\left(\frac{\delta^*}{\delta}\right)$.

(ii) The ratio of momentum thickness to boundary layer thickness $\left(\frac{\theta}{\delta}\right)$.

☺ ***Solution:***

Velocity distribution: $\frac{u}{U}=3\left(\frac{y}{\delta}\right)-2\left(\frac{y}{\delta}\right)^2$

(i) Ratio of displacement thickness to boundary layer thickness $\left(\frac{\delta^*}{\delta}\right)$

We know that, $\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy$

$$=\int_0^\delta \left(1-\left(\frac{3y}{\delta}-\frac{2y^2}{\delta^2}\right)\right)dy$$

$$=\left[y-\frac{3y^2}{2\delta}+\frac{2y^3}{3\delta^2}\right]_0^\delta=\left(\delta-\frac{3}{2}\frac{\delta^2}{\delta}+\frac{2}{3}\frac{\delta^3}{\delta^2}\right)$$

$$\delta^* = \left(\delta - \frac{3}{2}\delta + \frac{2}{3}\delta\right) = \frac{1}{6}\delta$$

$$\therefore \quad \frac{\delta^*}{\delta} = \frac{1}{6} \quad \text{Ar}$$

(ii) ***Ratio of momentum thickness to boundary layer thickness*** $\left(\frac{\theta}{\delta}\right)$:

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(3\frac{y}{\delta} - 2\frac{y^2}{\delta^2}\right)\left(1 - \left(\frac{3y}{\delta} - \frac{2y^2}{\delta^2}\right)\right) dy$$

$$= \int_0^\delta \left(\frac{3y}{\delta} - \frac{2y^2}{\delta^2}\right)\left(1 - \frac{3y}{\delta} + \frac{2y^2}{\delta^2}\right) dy$$

$$= \int_0^\delta \left(\frac{3y}{\delta} - \frac{9y^2}{\delta^2} + \frac{6y^3}{\delta^3} - \frac{2y^2}{\delta^2} + \frac{6y^3}{\delta^3} - \frac{4y^4}{\delta^4}\right) dy$$

$$= \int_0^\delta \left(\frac{3y}{\delta} - \frac{11y^2}{\delta^2} + \frac{12y^3}{4\delta^3} - \frac{4y^4}{\delta^4}\right) dy$$

$$= \left(\frac{3y^2}{2\delta} - \frac{11y^3}{3\delta^2} + \frac{12y^4}{4\delta^3} - \frac{4y^5}{5\delta^4}\right)_0^\delta$$

$$= \left(\frac{3}{2}\frac{\delta^2}{\delta} - \frac{11}{3}\frac{\delta^3}{\delta^2} + \frac{12\delta^4}{4\delta^3} - \frac{4\delta^5}{5\delta^4}\right)$$

$$\theta = \left(\frac{3}{2}\delta - \frac{11}{3}\delta + \frac{12}{4}\delta - \frac{4}{5}\delta\right) = \frac{1}{30}\delta$$

$$\frac{\theta}{\delta} = \frac{1}{30}$$

7.

2.14. DARCY'S EQUATION FOR LOSS OF HEAD DUE TO FRICTION IN PIPE

A *pipe* is a closed conduit through which the fluid flows under pressure. When the fluid flows through the piping system, some of the potential energy is lost due to friction. The various viscous friction effects associated with fluid are proportional to

- Length of pipe, L
- The wetted perimeter, P and
- V^n,

where V is the average velocity of flow and n is an index varying from 1.5 to 2.

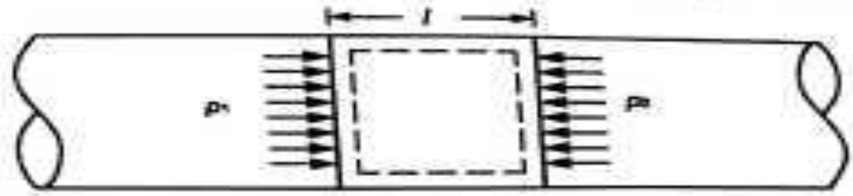


Figure 2.20

Consider a horizontal pipe of cross sectional area A carrying a fluid with a mean velocity v. Let p_1 and p_2 be the intensities of pressure at section 1 and 2 respectively. By applying Bernoulli's equation between the section 1 and 2,

$$\frac{v_1^2}{2g}+\frac{p_1}{w}+z_1=\frac{v_2^2}{2g}+\frac{p_2}{w}+z_2+h_f$$

Since $v_1 = v_2 = v$ and $z_1 = z_2$

$$\frac{p_1}{w}=\frac{p_2}{w}+h_f$$

$$\text{Loss of head, } h_f=\frac{p_1}{w}-\frac{p_2}{w}=\frac{p_1-p_2}{w}$$

Let f' be the frictional resistance per unit area at unit velocity,

Frictional resistance $= f' \times \text{Area} \times v^n$

$= f' \times PL \times v^n$

where, P = wetted perimeter of the pipe.

Pressure force of section 1 $= p_1A$

Pressure force of section 2 $= p_2A$

Resolving all forces horizontally,

$$p_1A = p_2A + \text{frictional resistance}$$

$$(p_1 - p_2)A = ff' \times PL \times v^n$$

$$(p_1 - p_2) = f' \times \frac{PL}{A} \times v^n$$

Dividing both side by specific weight w

$$\frac{p_1 - p_2}{w} = \frac{f'}{w} \times \frac{PL}{A} \times v^n$$

Substituting h_f values in above equation,

$$h_f = \frac{f'}{w} \times \frac{PL}{A} \times v^n$$

$$h_f = \frac{f'}{w} \times \left(\frac{P}{A}\right) \times Lv^n$$

$$h_f = \frac{f'}{w} \times \frac{Lv^n}{m} \quad \dots (1)$$

The ratio $\left(\frac{A}{P}\right)$ is called *Hydraulic Mean Depth* (H.M.D) or *Hydraulic radius* and represented by 'm'.

Hydraulic mean depth, $m = \left(\frac{A}{P}\right) = \frac{\pi/4 \times D^2}{\pi D} = \frac{D}{4}$

Substituting m value in equation (1)

$$h_f = \frac{f'}{w} \times \frac{Lv^n}{D/4}$$

Assume $n = 2$ for commercial pipes.

$$h_f = \frac{4f'Lv^2}{wD}$$

Multiplying both sides by '2g'

$$h_f = \frac{4f'Lv^2}{wD} \times \frac{2g}{2g}$$

$$h_f = \frac{2gf'}{w} \times \frac{4Lv^2}{2gD}$$

The term $\frac{2gf'}{w}$ is a non-dimensional quantity and Let us, replace by another constant f, then

$$h_f = f \times \frac{4Lv^2}{2gD}$$

$$\boxed{h_f = \frac{4fLv^2}{2gD}}$$

where, f – Darcy coefficient of friction

The above equation is called as *Darcy-weisbach equation*, sometimes, it may be written as

$$h_f = \frac{f_1 L v^2}{2gD}$$

where, f_1 = friction factor.

Darcy-weisbach equation is commonly used for computing the loss of head due to friction in pipes.

8.

A pipe 75mm diameter is 5m long and the velocity of flow of water in the pipe is 2.8m/s. if the central 2m length of pipe was replaced by 100mm diameter pipe and the change of section being sudden, calculate the loss of head and the corresponding power saving. Take f = 0.038 for the pipes of both diameters

Given data:

Diameter of small pipe, $D_1 = 75mm = 0.075m$

Diameter of large pipe, $D_2 = 100mm = 0.1m$

Velocity of flow in small pipe, $V_1 = 2.8m/s$

Co-efficient of friction, $f = 0.038$

☺ ***Solution:***

Velocity of flow in the larger diameter pipe,

$$A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{\frac{\pi}{4}\times D_1^2 \times V_1}{\frac{\pi}{4}\times D_2^2} = \frac{\frac{\pi}{4}\times(0.075)^2 \times 2.8}{\frac{\pi}{4}\times(0.1)^2} = 1.575m/s$$

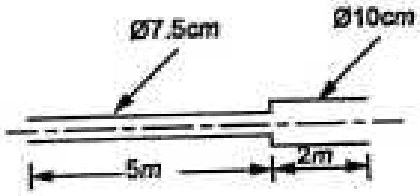


Figure 2.33

Head lost due to friction for 75mm diameter and 5m long pipe

$$h_f = \frac{4\,f\,LV^2}{2\,g\,D} = \frac{4\times0.038\times5\times(2.8)^2}{2\times9.81\times0.075} = 4.05m$$

When the central 2m length of the 75mm diameter pipe is replaced by a 10(diameter pipe, then the various head losses are as follows.

(i) Loss of head due to friction in the section 1-2:

$$h_{f_1} = \frac{4\,f\,L_1 V_1^2}{2\,g\,D_1} = \frac{4\times0.038\times1.5\times(2.8)^2}{2\times9.81\times0.075} = 1.215m$$

UNIT III

DIMENSIONAL ANALYSIS

PART – A

1. What are the methods of dimensional analysis

There are two methods of dimensional analysis. They are,
a. Rayleigh - Retz method
b. Buckingham's theotem method.
Nowadays Buckingham's theorem method is only used.

2. Describe the Rayleigh's method for dimensional analysis.

Rayleigh's method is used for determining the expression for a variable which depends upon maximum three or four variables only. If the number of independent variables becomes more than four, then it is very difficult to find the expression for dependent variable.

3. What do you mean by dimensionless number

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension or elastic force. As this is a ratio of one force to other force, it will be a dimensionless number.

4. Name the different forces present in fluid flow

Inertia force
Viscous force
Surface tension force
Gravity force

5. State Buckingham's Π theorem

It states that if there are 'n' variables in a dimensionally homogeneous equation and if these variables contain 'm' fundamental dimensions (M,L,T), then they are grouped into (n-m), dimensionless independent Π-terms.

6. State the limitations of dimensional analysis.

1. Dimensional analysis does not give any due regarding the selection of variables.
2. The complete information is not provided by dimensional analysis.
3. The values of coefficient and the nature of function can be obtained only by experiments or from mathematical analysis.

7. Define Similitude

Similitude is defined as the complete similarity between the model and prototype.

8. State Froude's model law

Only Gravitational force is more predominant force. The law states 'The Froude's number is same for both model and prototype'

9. What are the similarities between model and prototype?

(i) Geometric Similarity
(ii) Kinematicc Similarity
(iii) Dynamic Similarity

10. Define Weber number.

It is the ratio of the square root of the inertia force to the surface tension force.

PART-B

1.

Check the dimensional homogeneity for the equation v = u + ft.

Solution:

$$v = u + ft$$

Dimensions of each parameter,

v = Velocity (final) = LT^{-1}

u = Velocity (initial) = LT^{-1}

f = Acceleration = LT^{-2}

t = Time = T

So, the dimensional equation,

$$LT^{-1} = LT^{-1} + LT^{-2} \times T$$

$$LT^{-1} = LT^{-1} + LT^{-1}$$

$$= 2\,(LT^{-1})$$

The dimensions on both the sides are identical. So, the given equation is **dimensionally homogeneous.** **Ans.**

2.

Determine the dimension of the following quantities:

(i) Discharge (ii) Kinematic viscosity (iii) Force, and (iv) Specific volume

Solution:

Discharge = area × Velocity

$$= L^2 \times \frac{L}{T} = \frac{L^3}{T} = \boldsymbol{L^3 T^{-1}}$$

Kinematic viscosity $(\nu) = \frac{\mu}{\rho}$

where μ is given by, $\tau = \mu \frac{du}{dy}$

$$\mu = \frac{\tau}{du\ dy}$$

$$= \frac{\text{Shear stress}}{\frac{L}{T} \times \frac{1}{L}}$$

$$= \frac{force / area}{1/T}$$

$$= \frac{\text{Mass} \times \text{acceleration}}{\text{area} \times 1\ T}$$

$$= \frac{M \times L\ T^2}{L^2 \times 1\ T}$$

$$= \frac{M\ L}{L^2 \times T^2\ 1\ T}$$

$$= \frac{M}{LT} = \boldsymbol{M L^{-1} T^{-1}}$$

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3} = M L^{-3}$$

$$\therefore \text{Kinematic viscosity } (\nu) = \frac{\mu}{\rho} = \frac{M L^{-1} T^{-1}}{M L^{-3}} = \boldsymbol{L^2 T^{-1}}$$

Force = Mass × Acceleration

$$= M \times \frac{\text{Length}}{\text{Time}^2}$$

$$= \frac{ML}{T^2} = M\,L\,T^{-2}$$ **Ans.**

Specific weight $= \frac{\text{Weight}}{\text{Volume}} = \frac{\text{Force}}{\text{Volume}}$

$$= \frac{M\,LT^{-2}}{L^3} = M\,L^{-2}\,T^{-2}$$ **Ans.**

3.

Find an expression for the drag force on smooth sphere of diameter D, moving with uniform velocity v, in fluid density ρ and dynamic viscosity μ.

Solution:

The drag force F is a function of diameter D, velocity v, fluid density ρ and dynamic velocity μ.

Mathematically, $F = f(D, v, \rho, \mu)$ or

$$F = C\,(D^a, v^b, \rho^c, \mu^d) \quad \dots (1)$$

where C is a non-dimensional constant using M-L-T system. The corresponding equation for dimensions is

$$MLT^{-2} = C\,[L^a\,(L\,T^{-1})^b\,(ML^{-3})^c\,(ML^{-1}\,T^{-1})^d\,]$$

For dimensional homogeneity, the exponents of each dimension on both sides of the equation must be identical. Thus,

$$\text{For } M\text{: } 1 = c + d \quad \dots (i)$$

$$\text{For } L\text{: } 1 = a + b - 3c - d \quad \dots (ii)$$

$$\text{For } T\text{: } -2 = -b - d \quad \dots (iii)$$

There are four unknowns (a, b, c, and d) but three equations hence, it is not possible to find the values of a, b, c and d but three of them can be expressed in terms of fourth variable which is most important. But viscosity is a vital one and hence a, b, c are expressed in terms of d which is the power to viscosity.

... from (i)

$$\therefore c = 1 - d$$

$$b = 2 - d$$

$$a = 1 - b + 3c + d$$

$$= 1 - 2 + d + 3 - 3d + d = 2 - d$$

Putting these values of exponents in equation (i) we get

$$F = C\,[D^{2-d} \cdot v^{2-d} \cdot \rho^{1-d} \cdot \mu^{d}]$$

$$= C\,[\,D^2\, v^2\, \rho\, (D^{-d}\, V^{-d}\, \rho^{-d}\, \mu^{d})\,]$$

$$= C\left[\rho D^2 v^2 \left[\frac{\mu}{\rho v D}\right]^{d}\right]$$

$$F = \rho\, v^2\, D^2\, \phi\left[\frac{\mu}{\rho v D}\right]$$

4.

`

Efficiency η of a fan depends on the density ρ, the dynamic viscosity of the fluid μ, the angular velocity ω, diameter D of the rotor and the discharge Q. Express η in terms of dimensional parameters. *[AMIE]*

Solution:

The efficiency η of a fan is a function of:

(i) Density, ρ

(ii) Viscosity, μ

(iii) Angular velocity, ω

(iv) Diameter, D and

(v) Discharge, ϕ

Mathematically, $\eta = F\,(\rho, \mu, \omega, D, Q)$

$$\eta = c\,(\rho^a, \mu^b, \omega^c, D^d, Q^e) \quad \ldots (1)$$

where 'C' is a non-dimensional constant using M-L-T system, the corresponding equation for dimension is:

$$M^0 L^0 T^0 = C\,[(ML^{-3})^a\,(ML^{-1}T^{-1})^b\,(T^{-1})^c\,(L)^d\,(L^3T^{-1})^e]$$

For dimensional homogeneity, the exponents of each dimension on both sides of the equation must be identical. Thus,

For M: $0 = a + b$

For L: $0 = -3a - b + d + 3e$

For T: $0 = -b - c - e$

There are five variables and we have only three equations. Experience has shown that recognized dimensionless groups appear, if the exponents of D, ω and ρ are valuated in terms of b and e (exponents of viscosity and discharge which are more important).

$a = -b;$

$c = -(b+1);$

$d = 3a+b-3e = 3(-b)+b-3e = -(2b+3e)$

Substituting these values of exponents in equation (i) we get

$$\eta = C\,(\rho^{-b}\mu^{b}\omega^{-(b+e)}D^{-(2b+3e)}Q^{e})$$

$$= C\,(\rho^{-b}\mu^{b}\omega^{-b}\omega^{-e}D^{-2b}D^{-3e}Q^{e})$$

$$= C\left[\left(\frac{\mu}{\rho\omega D^2}\right)^{b}\left(\frac{Q}{\omega D^3}\right)^{e}\right]$$

$$= Q\left[\left(\frac{\mu}{\rho\omega D^2}\right),\left(\frac{Q}{\omega D^3}\right)\right]$$ **Ans.**

5.

Find an expression for the drag force on smooth sphere of diameter D moving with uniform velocity v in fluid density ρ and dynamic viscosity μ

Solution:

The functional relationship can be written as

$$F = f(D, v, \rho, \mu)$$

Again it can be written as

$$f_1 = (F, D, v, \rho, \mu) \quad \ldots (1)$$

The total number of variables, $n = 5$

Fundamental parameters, $m = 3$

So, the number of π-terms $= n - m = 5 - 3 = 2$

Therefore, the equation (1) can be written as

$$f_1(\pi_1, \pi_2) = 0 \quad \ldots. (2)$$

Each π-term has $m + 1$ variables

Here, D, v and ρ are selected as repeating variables.

So, $\pi_1 = D^{a_1} \times v^{b_1} \times \rho^{c_1} \times F$

$\pi_2 = D^{a_2} \times v^{b_2} \times \rho^{c_2} \times \mu$

Dimensions of each parameter are:

Force, $F = MLT^{-2}$

Diameter, $D = L$

Velocity, $v = LT^{-1}$

Density, $\rho = ML^{-3}$

Dynamic viscosity, $\mu = ML^{-1}T^{-1}$

π_1-term:

$$\pi_1 = D^{a_1} \times v^{b_1} \times \rho^{c_1} \times F$$

Substituting dimensions of each parameters/variables

$$M^0L^0T^0 = L^{a_1} \times (LT^{-1})^{b_1} \times (ML^{-3})^{C_1} \times MLT^{-2}$$

Comparing coefficient of each exponents on both sides,

For M; $0 = c_1 + 1$

For L; $0 = a_1 + b_1 - 3c_1 + 1$

For T; $0 = -b_1 - 2$

From (i), $c_1 = -1$

From (iii), $b_1 = -2$

Substituting values of c_1 & b_1 in equation (ii)

$$0 = a_1 - 2 - 3(-1) + 1$$

$$a_1 = 2 - 3 - 1$$

$$a_1 = -2$$

Now, $$\pi_1 = D^{-2} v^{-2} \rho^{-1} F = \frac{F}{\rho v^2 D^2}$$

π_2-term:

$$\pi_2 = D^{a_2} \times v^{b_2} \times \rho^{c_2} \times \mu$$

Substituting dimensions of each parameters/variables

$$M^0L^0T^0 = L^{a_2} \times (LT^{-1})^{b_2} \times (ML^{-3})^{c_2} \times ML^{-1}T^{-1}$$

Comparing coefficients of each exponent on both sides,

For M; $0 = c_2 + 1$

For L; $0 = a_2 + b_2 - 3c_2 - 1$

For T; $0 = -b_2 - 1$

From (iv), $c_2 = -1$

From (vi), $b_2 = -1$ and

Substituting values of c_2 and b_2 in (v)

$$0 = a_2 - 1 - 3(-1) - 1$$

$$\therefore \quad a_2 = 1 - 3 + 1 = -1$$

Now, $\pi_2 = D^{-1} \times v^{-1} \times \rho^{-1} \times \mu = \dfrac{\mu}{\rho v D}$

Substituting π_1 and π_2 in equation (2)

$$f_1\left(\frac{F}{\rho v^2 D^2}, \frac{\mu}{\rho v D}\right) = 0$$

$$f_1 - \frac{F}{\rho v^2 D^2} f\left(\frac{\mu}{\rho v D}\right)$$

$\therefore$ The drag force, $\boldsymbol{F = \rho v^2 D^2 \varphi\left(\dfrac{\mu}{\rho v D}\right)}$

6.

The efficiency η of a fan depends on the density ρ, the dynamic viscosity μ, angular velocity ω, diameter D of the motor and the discharge Q. Express the efficiency η in terms of dimensional parameters.

Solution:

The parameters involved in a given analysis are η, ρ, μ, ω, D and Q.

Dimensions of each parameter are

Efficiency, η = No dimensions

Density, $\rho = ML^{-3}$

Dynamic viscosity, $\mu = ML^{-1}T^{-1}$

Angular velocity, $\omega = T^{-1}$

Discharge, $Q = L^{-3}T^{-1}$

The function relationship can be written as

$$\eta = f(D, \rho, \omega, \mu, Q) \quad \dots (1)$$

Dimensionless equation is

$$M^0L^0T^0 = L^{a_2} \times \left(ML^{-3}\right)^{b_2} \times \left(T^{-1}\right)^{c_2} \times ML^{-1}T^{-1}$$

By comparison of coefficients of exponents on both sides

For M; $\quad 0 = b_2 + 1 \qquad \ldots \text{(iv)}$

For L; $\quad 0 = a_2 - 3b_2 - 1 \qquad \ldots \text{(v)}$

For T; $\quad 0 = -c_2 - 1 \qquad \ldots \text{(iv)}$

From (iv) $\quad b_2 = -1$

From (vi) $\quad c_2 = -1$

Substituting b_2 and c_2 in (v)

$$0 = a_2 - 3(-1) - 1$$

$$a_2 = -3 + 1 = -2$$

Now, $\quad \pi_2 = D^{-2} \times \rho^{-1} \times \omega^{-1} \times \mu = \dfrac{\mu}{\rho D^2 \omega}$

π_3 – term:

$$\pi_3 = D^{a_3} \times \rho^{b_3} \times \omega^{c_3} \times Q$$

Dimensionless equation is given by

$$M^0L^0T^0 = L^{a_3} \times \left(ML^{-3}\right)^{b_3} \times \left(T^{-1}\right)^{c_3} \times L^3T^{-1}$$

By exponents coefficients comparison

For M; $\quad 0 = b_3 \qquad \ldots \text{(vii)}$

For L; $\quad 0 = a_3 - 3b_3 + 3 \qquad \ldots \text{(viii)}$

For T; $\quad 0 = -c_3 - 1 \qquad \ldots \text{(ix)}$

From (vii), $\quad b_3 = 0$

From (ix), $\quad c_3 = -1$

Substituting b_3 and c_3 in (viii)

$$0 = a_3 - 3 \times 0 + 3$$

$\therefore \quad a_3 = -3$

Now, $\quad \pi_3 = D^{-3} \times \rho^0 \times \omega^{-1} \times Q = \dfrac{Q}{D^3 \omega}$

Substituting π_1, π_2 and π_3 in equation (3)

$$f_1\left(\eta, \frac{\mu}{\rho D^2 \omega}, \frac{Q}{D^3 \omega}\right) = 0$$

$$\therefore \quad \text{Efficiency } \eta = \varphi\left(\frac{\mu}{\rho D^2 \omega}, \frac{Q}{D^3 \omega}\right)$$

7.

The pressure difference Δp in a pipe of diameter D and length l due to turbulent flow depends on the velocity v, viscosity μ, density ρ and roughness K. Using Buckingham's π-theorem, obtain an expression for Δp.

☺ ***Solution:***

The functional relationship can be written as

$$\Delta p = f(D, l, v, \mu, \rho, K)$$

$$f_1(\Delta p, D, l, v, \mu, \rho, k) = 0 \qquad \text{....(1)}$$

Number of variables $n = 7$

Number of 'π' terms $= 7 - 3 = 4$terms

$$\therefore f_2(\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

Each π terms contains $(m+1)$ variables choosing D, v and f as repeating variables get

$$\pi = D^{a1}.v^{b1}.\rho^{c1}.\Delta p$$

$$\pi_2 = D^{a2}.v^{b2}.\rho^{c2}.l$$

$$\pi_3 = D^{a3}.v^{b3}.\rho^{c3}.\mu$$

$$\pi_4 = D^{a4}.v^{b4}.\rho^{c4}.K$$

π_1 term:

$$\pi_1 = M^0 L^0 T^0 = L^{a1}(LT^{-1})^{b1}.(ML^{-3})^{c1}(ML^{-1}T^{-2})$$

Equating the exponent of $M.L$ and T, we get

For M; $\quad 0 = c_1 + 1 \Rightarrow c_1 = -1$

For L; $\quad 0 = a_1 + b_1 - 3c_1 - 1$

For T; $\quad 0 = -b_1 - 2 \quad \therefore b_1 = 2$

$$0 = a_1 - 2 + 3 - 1 \quad \therefore a_1 = 0$$

Substituting these values in π_1 term, we get

$$\pi_1 = D^o\, v^{-2}\, \rho^{-1}\, \Delta p = \frac{\Delta p}{\rho V^2}$$

π_2 term:

$$\pi_2 = M^o L^o T^o = L^{a_2} (LT^{-1})^{b_2} . (ML^{-3})^{c_2} L$$

Equating the exponent of M L and T, we get

For M; $0 = c_2$

For L, $0 = a_2 + b_2 - 3c_2 + 1$

For T; $0 = -b_2$

$$b_2 = c_2 = 0$$

$$\therefore \quad a_2 = -1$$

$$\pi_2 = D^{-1}\, v^o \rho^o\, l$$

$$\pi_2 = l/D$$

π_3 term:

$$\pi_3 = M^o L^o T^o = L^{a_3} (LT^{-1})^{b_3} . (ML^{-3})^{c_3} M L^{-1} T^{-1}$$

Equating the exponent of $M.L$ and T we get

For M; $0 = c_3 + 1 \Rightarrow c_3 = -1$

For L; $0 = a_3 + b_3 - 3c_3 - 1$

For T; $0 = -b_3 - 1 \Rightarrow b_3 = -1$

$$\therefore a_3 = -1$$

$$\pi_3 = D^{-1}\, v^{-1} \rho^{-1}\, \mu = \frac{\mu}{D v \rho}$$

π_4 term:

$$\pi_4 = M^o L^o T^o = L^{a_4} (LT^{-1})^{b_4} . (ML^{-3})^{c_4} L \quad (\because \text{Dimension for } K = L)$$

For M; $0 = c_4$

For L; $0 = a_4 + b_4 - 3c_4 + 1$

For T: $0 = -b_4$

$$\therefore a_4 = -1$$

$$\pi_4 = D^{-1}v^0\rho^0K$$

$$\pi_4 = \frac{K}{D}$$

Substituting these values in equation (2), we get

$$f_2 = (\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

$$f_2\left(\frac{\Delta p}{\rho v^2}, \frac{l}{D}, \frac{\mu}{Dv\rho}, \frac{K}{D}\right) = 0$$

$$\frac{\Delta p}{\rho v^2} = \phi\left(\frac{l}{D}, \frac{\mu}{Dv\rho}, \frac{K}{D}\right)$$

$$\Delta p = \rho v^2 \varphi\left(\frac{l}{D}, \frac{\mu}{Dv\rho}, \frac{K}{D}\right)$$

UNIT-IV
PUMPS
PART – A (2 Marks)

1. What is meant by Pump?

A pump is device which converts mechanical energy into hydraulic energy.

2. Define a centrifugal pump

If the mechanical energy is converted into pressure energy by means of centrifugal force cutting on the fluid, the hydraulic machine is called centrifugal pump.

3. Define suction head (hs).

Suction head is the vertical height of the centre lines of the centrifugal pump above the water surface in the tank or pump from which water is to be lifted. This height is also called suction lift and is denoted by hs.

4. Define delivery head (hd).

The vertical distance between the center line of the pump and the water surface in the tank to which water is delivered is known as delivery head. This is denoted by hd.

5. Define static head (Hs).

The sum of suction head and delivery head is known as static head. This is represented by 'Hs' and is written as, Hs = hs+ hd

6. Mention main components of Centrifugal pump.

i) Impeller ii) Casing

iii) Suction pipe,strainer & Foot valve iv) Delivery pipe & Delivery valve

7. What is meant by Priming?

The delivery valve is closed and the suction pipe, casing and portion of the delivery pipe upto delivery valve are completely filled with the liquid so that no air pocket is left. This is called as priming.

8. Define Manometric head.

It is the head against which a centrifugal pump work.

9. Describe multistage pump with

a. impellers in parallel b. impellers in series. In multi stage centrifugal pump,

a. when the impellers are connected in series (or on the same shaft) high head can be developed.

b. When the impellers are in parallel (or pumps) large quantity of liquid can be discharged.

10.. Define specific speed of a centrifugal pump (Ns).

The specific speed of a centrifugal pump is defined as the speed of a geometrically circular pump which would deliver one cubic meter of liquid per second against a head of one meter. It is denoted by 'Ns'.

11. What do you understand by characteristic curves of the pump?

Characteristic curves of centrifugal pumps are defined those curves which are plotted from the results of a number of tests on the centrifugal pump.

12. Why are centrifugal pumps used sometimes in series and sometimes in parallel?

The centrifugal pumps used sometimes in series because for high heads and in parallel for high discharge

13.Define Mechanical efficiency.

I

t is defined as the ratio of the power actually delivered by the impeller to the power supplied to the shaft.

14. Define overall efficiency.

It is the ratio of power output of the pump to the power input to the pump.

15. Define speed ratio, flow ratio.

Speed ratio: It is the ratio of peripheral speed at outlet to the theoretical velocity of jet corresponding to manometric head.

Flow ratio: It is the ratio of the velocity of flow at exit to the theoretical velocity of jet corresponding to manometric head.

16.. Mention main components of Reciprocating pump.

Piton or Plunger

Suction and delivery pipe

Crank and Connecting rod

17.. Define Slip of reciprocating pump. When the negative slip does occur?

The difference between the theoretical discharge and actual discharge is called slip of the pump.

But in sometimes actual discharge may be higher then theoretical discharge, in such a case coefficient of discharge is greater then unity and the slip will be negative called as negative slip.

18. What is indicator diagram?

Indicator diagram is nothing but a graph plotted between the pressure head in the cylinder and the distance traveled by the piston from inner dead center for one complete revolution of the crank

19. What is meant by Cavitations?

It is defined phenomenon of formation of vapor bubbles of a flowing liquid in a region where the pressure of the liquid falls below its vapor pressure and the sudden collapsing of theses vapor bubbles in a region of high pressure.

20. What are rotary pumps?

Rotary pumps resemble like a centrifugal pumps in appearance. But the working method differs. Uniform discharge and positive displacement can be obtained by using these rotary pumps, It has the combined advantages of both centrifugal and reciprocating pumps.

The impeller of a centrifugal pump has external and internal diameters 500mm and 250mm respectively, width of outlet 50mm and running of 1200rpm. It works against a head of 48m. The velocity of flow through the impeller is constant and equal to 3.0m/s. The vanes are set back at an angle of 40° at outlet. Determine

(i) Inlet vane angle
(ii) Work done by the impeller on water per second and
(iii) Manometric efficiency.

Given data:

External diameter, $D_2 = 500mm = 0.5m$

Internal diameter, $D_1 = 250mm = 0.25m$

Width of outlet, $B_2 = 50mm = 0.05m$

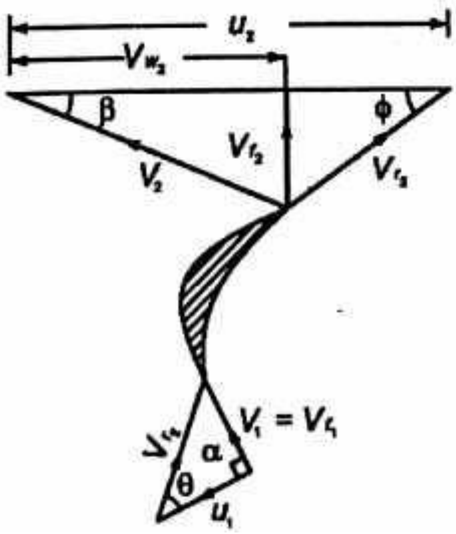
Speed, $N = 1200rpm$

Head, $H_m = 48m$

Velocity of flow, $V_{f_1} = V_{f_2} = 3m/s$

Vane angle at outlet, $\phi = 40°$

☺ ***Solution:***



Tangential velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1200}{60} = 15.7 m/s$$

From inlet velocity triangle,

$$\tan\theta = \frac{V_{f_1}}{u_1} = \frac{3}{15.7} = 0.191$$

$$\theta = \tan^{-1}(0.191) = \mathbf{10.81^\circ}$$ **Ans.**

Discharge, $Q = \pi D_2 B_2 V_{f_2} = \pi \times 0.5 \times 0.05 \times 3 = 0.2356 m^3/s$

Tangential velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1200}{60} = 31.41 m/s$$

From outlet velocity triangle,

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\tan 40^\circ = \frac{3}{31.41 - V_{w_2}}$$

$$31.41 - V_{w_2} = \frac{3}{\tan 40^\circ} = 3.575$$

$$V_{w_2} = 31.41 - 3.575 = 27.835 m/s$$

Work done by the impeller per second,

$$= \frac{wQ}{g} \times V_{w_2} u_2$$

$$= \frac{9.81 \times 0.2356}{9.81} \times 27.835 \times 31.41 \qquad \text{where, } w = 9.81 kN/m^3$$

Ans.

$$= \mathbf{205.98\ kN\text{-}m}$$

Manometric efficiency, $\eta_{mano} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 48}{27.835 \times 31.41} = 0.5386 = \mathbf{53.86\%}$ **Ans.**

2.

A centrifugal pump runs at 1000rpm with their vane angles at inlet and outlet a 20° and 35° respectively. The internal and external diameters are 25cm and 50cm respectively. Find the work done per N of water assuming velocity of flow a constant. Water enters radially through the pump.

Given data:

Speed, $N = 1000rpm$.

Inlet vane angle, $\theta = 20°$

Outlet vane angle, $\phi = 35°$

Internal diameter, $D_1 = 25cm = 0.25m$

External diameter, $D_2 = 50cm = 0.5m$

Radial entry, $V_{w_1} = 0$

Velocity of flow $= V_{f_1} = V_{f_2}$ = constant

☺ ***Solution:***

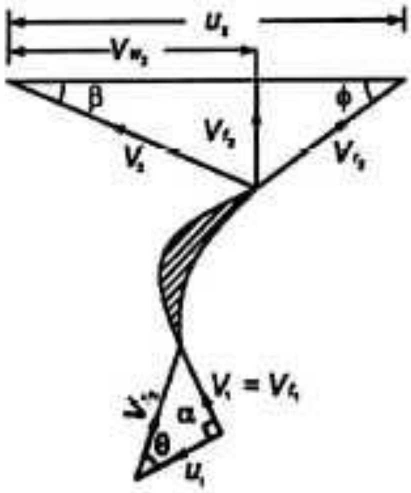


Figure 4.20

Tangential velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.1 m/s$$

Tangential velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1000}{60} = 26.2 m/s$$

From inlet velocity triangle,

$$\tan\theta = \frac{V_{f_1}}{u_1}$$

$$\tan 20^\circ = \frac{V_{f_1}}{13.1}$$

$$V_{f_1} = 13.1 \times \tan 20^\circ = 4.768 m/s$$

$$V_{f_1} = V_{f_2} = 4.768 m/s.$$

From outlet velocity triangle,

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\tan 35^\circ = \frac{4.768}{26.2 - V_{w_2}}$$

$$26.2 - V_{w_2} = \frac{4.768}{\tan 35^\circ} = 6.8$$

$$V_{w_2} = 26.2 - 6.8 = 19.4 m/s$$

Work done per N of water $= \dfrac{V_{w_2} u_2}{g} = \dfrac{19.4 \times 26.2}{9.81} = \mathbf{51.8 N\text{-}m}$

3.

A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1200rpm works against a total head of 32m. The velocity of flow through the impeller is constant and equal to 3m/s. The vanes are set back at an angle of 30° at the outlet. If the outer diameter of the impeller is 600mm and width at outlet is 50mm, determine (a) vane angle at inlet, (b) work done per second by impeller, and (c) manometric efficiency.

Given data:

$D_2 = 2D_1$

Speed, $N = 1200rpm$

Head, $H_m = 32m$

Velocity of flow, $V_{f_1} = V_{f_2} = 3m/s$

Vane angle at outlet, $\phi = 30°$

Outer diameter of the impeller, $D_2 = 600mm = 0.6m$

Width at outlet, $B_2 = 50mm = 0.5m$

Solution:

Inlet diameter of the impeller,

$$D_1 = \frac{D_2}{2} = \frac{0.6}{2} = 0.3m$$

Tangential velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.3 \times 1200}{60} = 18.85m/s$$

Tangential velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 1200}{60} = 37.7m/s$$

Vane angle at inlet,

$$\tan\theta = \frac{V_{f_1}}{u_1} = \frac{3}{18.85} = 0.159$$

$$\theta = \tan^{-1}(0.159) = 9°$$

Discharge, $Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.6 \times 0.05 \times 3 = 0.2827m^3/s$

From outlet velocity triangle,

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\tan 30° = \frac{3}{18.85 - V_{w_2}}$$

$$18.85 - V_{w_2} = \frac{3}{\tan 30°} = 5.196$$

$$V_{w_2} = 18.85 - 5.196$$

$$V_{w_2} = 13.65m/s$$

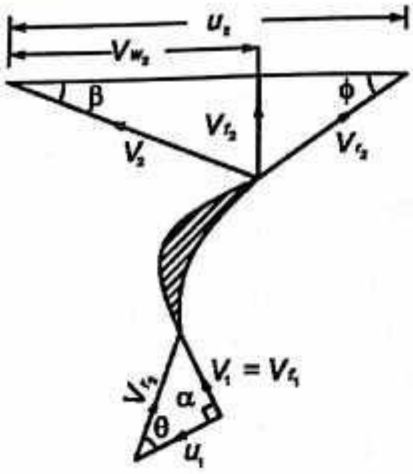


Figure 4.24

Work done per second by the impeller,

$$= \frac{W}{g} V_{w_2} u_2 = \frac{\rho g Q}{g} \times V_{w_2} u_2$$

$$= \frac{1000 \times 9.81 \times 0.2827}{9.81} \times 13.65 \times 37.7 = \mathbf{145478.8 m/s} \text{ Ans.}$$

Manometric efficiency,

$$\eta_{mano} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 32}{13.65 \times 37.7} = 0.61 = \mathbf{61\%} \qquad \text{Ans.}$$

4.

A three stage centrifugal pump has impellers 500mm in diameter and 60mm width at outlet. All the impellers are keyed to the same shaft. The vanes of each impeller are having outlet angle as 40°. The speed of the pump is 400rpm and the total manometric head developed is 25m. If the discharge through the pump is 0.12m³/s. Find the manometric efficiency.

Given data:

Number of pumps, $n = 3$

Diameter of impeller at outlet, $D_2 = 500mm = 0.5m$

Width at outlet, $B_2 = 60mm = 0.06m$

Outlet vane angle, $\phi = 40°$

Speed, $N = 400 rpm$

Manometric head, $H_m = 25m$

Discharge, $Q = 0.12 m^3/s$

Solution:

Manometric head for each stage,

$$H_m = \frac{25}{3} = 8.333m$$

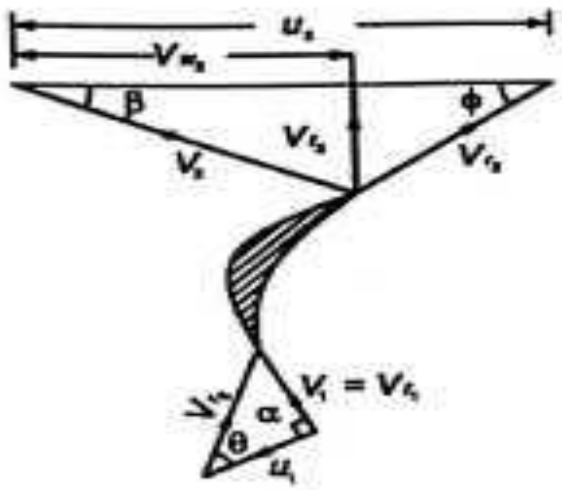


Figure 4.25

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 400}{60} = 10.47 m/s$$

Discharge, $Q = \pi\, D_2\, B_2\, V_{f_2}$

$$0.12 = \pi \times 0.5 \times 0.06 \times V_{f_2}$$

$$V_{f_2} = 1.273 m/s$$

From outlet velocity triangle,

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\tan 40° = \frac{1.273}{10.47 - V_{w_2}}$$

$$10.47 - V_{w_2} = \frac{1.273}{\tan 40^\circ} = 1.517$$

$$V_{w_2} = 10.47 - 1.517 = 8.95 m/s$$

Manometric efficiency,

$$\eta_{mano} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 8.33}{8.95 \times 10.47} = 0.8712 = \mathbf{87.12\%}$$ **Ans.**

5.

A single acting reciprocating pump, running at 60rpm delivers 0.53m³ of water per minute. The diameter of the piston is 200mm and stroke length 300mm. The suction and delivery heads are 4m and 12m respectively. Determine

- ***(i) Theoretical discharge.***
- ***(ii) Co-efficient of discharge.***
- ***(iii) Percentage slip of the pump, and***
- ***(iv) Power required to run the pump.***

Given data:

Speed, $N = 60 rpm$

Discharge, $Q_{act} = 0.53 m^3/min = \frac{0.53}{60} = 8.83 \times 10^{-3} m^3/s$

Diameter of piston, $D = 200mm = 0.2m$

Stroke length, $L = 300mm = 0.3m$

Suction head, $h_s = 4m$

Delivery head, $h_d = 12m$

☺ ***Solution:***

$$\text{Theoretical discharge, } Q_{th} = \frac{ALN}{60} = \frac{\frac{\pi}{4}\times 0.2^2 \times 0.3 \times 60}{60}$$

$$= 9.42 \times 10^{-3}\, m^3/s$$ **Ans.**

$$\text{Co-efficient of discharge, } C_d = \frac{Q_{act}}{Q_{th}} = \frac{8.83\times 10^{-3}}{9.42\times 10^{-3}} = 0.937$$ **Ans.**

$$\%\ \text{slip of the pump} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100$$

$$= \frac{9.42\times 10^{-3} - 8.83\times 10^{-3}}{8.83\times 10^{-3}} \times 100$$

$$= 6.68\%$$ **Ans.**

Power required to run the pump,

$$P = \frac{wALN}{60}\,(h_s + h_d)$$

$$= \frac{9.81\times \frac{\pi}{4}\times 0.2^2 \times 0.3 \times 60}{60}(4 + 12)\ \left(\because w = 9.81 kN/m^2\right)$$

$$= 1.479 kW$$ **Ans.**

6.

A double acting reciprocating pump, running at 50rpm is discharging 900 liters of water per minute. The pump has a stroke of 400mm. The diameter of piston is 250mm. The delivery and suction heads are 25m and 4m respectively. Find the slip of the pump and power required to drive the pump.

Given data:

Speed, $N = 50rpm$

Discharge, $Q = 900 lit/min = \frac{900}{1000 \times 60} = 0.015 m^3/s$

Strokes, $L = 400mm = 0.4m$

Diameter of piston, $D = 250mm = 0.25m$

Delivery head, $h_d = 25m$

Suction head, $h_s = 4m$

Solution:

Theoretical discharge,

$$Q_{th} = \frac{2ALN}{60} = \frac{2 \times \frac{\pi}{4} \times 0.25^2 \times 0.4 \times 50}{60} = 0.03272 m^3/s$$

$$\text{Slip} = Q_{th} - Q_{act} = 0.03272 - 0.015 = \mathbf{0.01772 m^3/s}$$ **Ans.**

Power required to drive the double acting pump,

$$P = \frac{2wALN}{60}(h_s + h_d)$$

$$= \frac{2 \times 9.81 \times \frac{\pi}{4} \times 0.25^2 \times 0.4 \times 50}{60}(25 + 4) \qquad (\because w = 9.81 kN/m^3)$$

$$= \mathbf{9.3 kW}$$ **Ans.**

7.

A 'three throw' pump has cylinders of 350mm diameter and stoke of 600mm each. The pump is required to deliver 0.12m³/s at a head of 100m. Frictional losses are estimated to be 2m in suction pipe and 22m in delivery pipe. Velocity of water in delivery pipe is 1m/s. Overall efficiency is 80% and the slip is 4.25%. Determine (i) Speed of the pump and (ii) Power required to run the pump.

Given data:

Three throw pump means three stages pump.

Diameter of cylinder, $D = 350mm = 0.35m$

Stroke, $L = 600mm = 0.6m$

Actual discharge, $Q_{act} = 0.12m^3/s$

Delivery head, $h_d = 100m$

Frictional losses in the suction pipe, $h_{fs} = 2m$

Frictional losses in the delivery pipe, $h_{fd} = 22m$

Velocity of water in delivery pipe, $V_d = 1m/s$

Overall efficiency, $\eta_o = 80\% = 0.8$

Slip = 4.25%

Solution:

For a three throw pump, the theoretical discharge is

$$Q_{th} = 3 \times \frac{ALN}{60} = 3 \times \frac{\frac{\pi}{4} \times (0.35)^2 \times 0.6 \times N}{60}$$

$$Q_{th} = 2.886 \times 10^{-3} \times N$$

$$\% \text{ slip} = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$4.25 = \left(1 - \frac{0.12}{2.886 \times 10^{-3} \times N}\right) \times 100$$

$$1 - \frac{0.12}{2.886 \times 10^{-3} \times N} = 0.0425$$

$$\frac{0.12}{2.886 \times 10^{-3} \times N} = 0.9575$$

$$N = \frac{0.12}{2.886 \times 10^{-3} \times 0.9575} = 43.42rpm \qquad \textbf{Ans.}$$

Total head against which pump has to work

$$H = (h_s + h_d) + (h_{fs} + h_{fd}) + \frac{V_d^2}{2g}$$

$$= 100 + 2 + 22 + \frac{1^2}{2 \times 9.81} = 124.05m$$

Water power = $w\, Q_{act}\, H$

$$= 9.81 \times 0.12 \times 124.05 \qquad (\because w = 9.81 kN/m^3)$$

$$= 146.03 kW$$

$$\text{Overall efficiency, } \eta_o = \frac{Water\ power}{Shaft\ power}$$

$$0.8 = \frac{146.03}{Shaft\ power}$$

Shaft power, $P =$ **182.54kW** **Ans.**

8.

The length and diameter of a suction pipe of a single acting reciprocating pump are 5m and 10cm respectively. The pump has a plunger of diameter 150mm and of stroke length of 300mm. The centre of the pump is 4m above water surface in the pump. The atmospheric pressure head is 10.3m of water and pump is running at 40rpm.

Determine:

- *(i) Pressure head due to acceleration at the beginning of the suction stroke.*
- *(ii) Maximum pressure head due to acceleration.*
- *(iii) Pressure head in the cylinder at the beginning and at the end of the stroke.*

Given data:

$l_s = 5m$

$d_s = 10cm = 0.1m$

$D = 150mm = 0.15m$

$L = 300mm = 0.3m$

$\therefore\ r = \frac{L}{2} = \frac{0.3}{2} = 0.15m$

$h_s = 4m$

$H_{atm} = 10.3m$ of water

$N = 40rpm$

Solution:

Acceleration head on the suction stroke,

$$h_{a_s} = \frac{l_s}{g}\frac{A}{a_s}\omega^2 r\cos\theta$$

At the beginning of the stroke, θ= 0°

$$\therefore h_{a_s} = \frac{l_s}{g}\frac{A}{a_s}\omega^2 r \qquad [\because \cos 0^\circ = 1]$$

$$= \frac{5}{9.81} \times \frac{\frac{\pi}{4}\times(0.15)^2}{\frac{\pi}{4}\times(0.1)^2} \times \left(\frac{2\pi\times 40}{60}\right)^2 \times 0.15 = \mathbf{3.02m\ of\ water}$$ **Ans.**

Maximum pressure head due to acceleration,

$$= h_s + h_{a_s} = 4 + 3.02 = 7.02m \text{ of water}$$

Absolute pressure head at the beginning of the stroke,

$$= H_{atm} - (h_s + h_{a_s}) = 10.3 - 7.02 = \mathbf{3.28m\ of\ water}$$ **Ans.**

Acceleration pressure head at the end of suction stroke,

$$h_{a_s} = \frac{-l_s}{g}\frac{A}{a_s}\omega^2 r \qquad [\because \theta = 180° \Rightarrow \cos 180° = -1]$$

$$= -3.02m \text{ of water}$$

Pressure head at the end of stroke $= H_{atm} - (h_s + h_{a_s}) = 10.3 - (4 - 3.02)$

$$= 9.32m \text{ of water} \qquad \textbf{Ans.}$$

UNIT-V
TURBINES
PART – A

1. **Define hydraulic machines.**
Hydraulic machines which convert the energy of flowing water into mechanical energy.

2. **Give example for a low head, medium head and high head turbine.**
Low head turbine – Kaplan turbine
Medium head turbine – Modern Francis turbine
High head turbine – Pelton wheel

3. **What is impulse turbine? Give example.**
In impulse turbine all the energy converted into kinetic energy. From these the turbine will develop high kinetic energy power. This turbine is called impulse turbine. Example: Pelton turbine

4. **What is reaction turbine? Give example.**
In a reaction turbine, the runner utilizes both potential and kinetic energies. Here portion of potential energy is converted into kinetic energy before entering into the turbine.
Example: Francis and Kaplan turbine.

5. **What is axial flow turbine?**
In axial flow turbine water flows parallel to the axis of the turbine shaft. Example:
Kaplan turbine

6. **What is mixed flow turbine?**
In mixed flow water enters the blades radially and comes out axially, parallel to the turbine shaft. Example: Modern Francis turbine.

7. **What is the function of spear and nozzle?**
The nozzle is used to convert whole hydraulic energy into kinetic energy. Thus the nozzle delivers high speed jet. To regulate the water flow through the nozzle and to obtain a good jet of water spear or nozzle is arranged.

8. **Define gross head and net or effective head.**
Gross Head: The gross head is the difference between the water level at the reservoir and the level at the tailstock.
Effective Head: The head available at the inlet of the turbine.

9. Define hydraulic efficiency.

It is defined as the ratio of power developed by the runner to the power supplied by the water jet.

10. Define mechanical efficiency.

It is defined as the ratio of power available at the turbine shaft to the power developed by the turbine runner.

11. Define volumetric efficiency.

It is defied as the volume of water actually striking the buckets to the total water supplied by the jet.

12. Define over all efficiency.

It is defined as the ratio of power available at the turbine shaft to the power available from the water jet.

13. Define the terms

(a) Hydraulic machines (b) Turbines (c) Pumps.

a. Hydraulic machines:

Hydraulic machines are defined as those machines which convert either hydraulic energy into mechanical energy or mechanical energy into hydraulic energy.

b. Turbines;

The hydraulic machines which convert hydraulic energy into mechanical energy are called turbines.

c. Pumps:

The hydraulic Machines which convert mechanical energy into hydraulic energy are called pumps.

14. What do you mean by gross head?

The difference between the head race level and tail race level when no water is flowing is known as gross head. It is denoted by Hg.

15. What do you mean by net head?

Net head is also known as effective head and is defined as the head available at the inlet of te turbine. It is denoted as H

16. What is draft tube? why it is used in reaction turbine?

The pressure at exit of runner of a reaction turbine is generally less than the atmospheric pressure. The water at exit cannot be directly discharged to tail race. A tube or pipe of gradually increasing area is used for discharging water from exit of turbine to tail race. This tube of increasing area is called draft tube.

17. What is the significance of specific speed?

Specific speed plays an important role for selecting the type of turbine. Also the performance of turbine can be predicted by knowing the specific speed of turbine.

18.. What are unit quantities?

Unit quantities are the quantities which are obtained when the head on the turbine is unity. They are unit speed, unit power unit discharge.

19. Why unit quantities are important

If a turbine is working under different heads, the behavior of turbine can be easily known from the values of unit quantities.

20. What do you understand by characteristic curves of turbine?

Characteristic curves of a hydraulic turbine are the curves, with the help of which the exact behavior and performance of turbine under different working conditions can be known.

21. Define the term 'governing of turbine'.

Governing of turbine is defined as the operation by which the speed of the turbine is kept constant under all conditions of working. It is done by oil pressure governor.

22. What are the types of draft tubes?

The following are the important types of draft tubes which are commonly used.

a. Conical draft tubes

b. Simple elbow tubes

c. Moody spreading tubes and

d. Elbow draft tubes with circular inlet and rectangular outlet.

A Pelton wheel having semi-circular buckets functions under a head of 150m and consumes 0.06m³/s of water. If 750mm diameter wheel turns 800rpm, calculate the power available at the nozzle and hydraulic efficiency of the wheel. Assume the coefficient of velocity as unity.

Given data:

Net head, $H = 150m$

Discharge, $Q = 0.06m^3/s$

Diameter of wheel, $D = 750mm = 0.75m$

Speed of wheel, $N = 800rpm$

Coefficient of velocity, $C_v = 1$

☺ ***Solution:***

Power available at the nozzle, $P = w\,Q\,H = 9.81 \times 0.06 \times 150 =$ **88.29*kW*** **Ans.**

Velocity of jet, $V_1 = C_v\sqrt{2\,g\,H}$

$$= 1 \times \sqrt{2 \times 9.81 \times 150} = 54.25m/s$$

Tangential velocity of the wheel, $u = \dfrac{\pi\,D\,N}{60} = \dfrac{\pi \times 0.75 \times 800}{60} = 31.4m/s$

Since, buckets are semi-circular, theoretically the jets get deflected through 180° and therefore, the blade angle $\phi = 0°$.

Hydraulic efficiency, $\eta_h = \dfrac{2u\,(V_1 - u)\,(1 + Cos\phi)}{V_1^2}$

$$= \frac{2 \times 31.4\,(54.25 - 31.4)\,(1 + Cos0)}{(54.25)^2}$$

$\eta_h = 0.9751 =$ **97.51%** **Ans.**

2.

A Pelton wheel turbine runs under a head of 400m at a speed of 1000rpm. It develops a power of 5000kW. Find the least diameter of jet and the pitch circle diameter of wheel. Assume overall efficiency of turbine as 85%; C_v = 0.99 and speed ratio is 0.45. Also find the number of buckets. Assume any data, if required.

Given data:

Head, $H = 400m$

Speed, $N = 1000rpm$

Power, $P = 5000kW$

Overall efficiency $\eta_o = 85\%$

$C_v = 0.99$

Speed ratio, $K_u = 0.45$

☺ ***Solution:***

Velocity of jet, $V_1 = C_v \sqrt{2gH} = 0.99 \times \sqrt{2 \times 9.81 \times 400} = 87.7 m/s$

Velocity of wheel, $u = K_u \times V_1 = K_u \sqrt{2gH}$

$= 0.45 \times \sqrt{2 \times 9.81 \times 400} = 39.87 m/s$

Velocity of wheel, $u = \frac{\pi D N}{60}$

$39.87 = \frac{\pi \times D \times 1000}{60}$

Diameter of wheel, $D = \mathbf{0.761m}$ **Ans.**

Overall efficiency, $\eta_o = \frac{P}{wQH}$

$0.85 = \frac{5000}{9.81 \times Q \times 400}$

$Q = 1.5 m^3/s$

We know that, Discharge, Q = Area of jet × Velocity of jet

$Q = \frac{\pi}{4} d^2 \times V_1$

$1.5 = \frac{\pi}{4} d^2 \times 87.7$

$d^2 = 0.02178$

$d = \mathbf{0.1476m}$ **Ans.**

Number of buckets, $Z = 15 + \frac{D}{2d} = 15 + \frac{0.761}{2 \times 0.1476} = \mathbf{17.58 \approx 18}$ **Ans.**

3.

A Pelton wheel is to develop 13,250kW under a net head of 800m while running at a speed of 600rpm. If the co-efficient of jet=0.97, speed ratio = 0.46 and the ratio of jet diameter is 1/15 of wheel diameter. Calculate (a) number of jets (b) Diameter of jets (c) Diameter of pitch circle (d) Quantity of water supplied to wheel. Assume overall efficiency as 85%.

Given data:

Power, $P = 13250kW$

Head, $H = 800m$

Speed, $N = 600rpm$

$C_v = 0.97$

Speed ratio, $K_u = 0.46$

$$\frac{d}{D} = \frac{1}{15}$$

$\eta_o = 85\%$

Solution:

$$\text{Velocity of jet, } V_1 = C_v\sqrt{2gH} = 0.97 \times \sqrt{2\times9.81\times800} = 121.53m/s$$

$$\text{Velocity of wheel, } u = K_u\sqrt{2gH} = 0.46 \times \sqrt{2\times9.81\times800} = 57.63m/s$$

$$\text{We know that, } u = \frac{\pi DN}{60}$$

$$57.63 = \frac{\pi\times D\times600}{60}$$

$$D = 1.834m \qquad \text{Ans.}$$

Ratio of jet diameter to wheel diameter

$$\frac{d}{D} = \frac{1}{15}$$

$$d = \frac{1}{15}\times D = \frac{1}{15}\times 1.834 = 0.122m \qquad \text{Ans.}$$

$$\text{Overall efficiency, } \eta_o = \frac{P}{wQH}$$

$$0.85 = \frac{13250}{9.81\times Q\times800}$$

$$Q = 1.986m^3/s \qquad \text{Ans.}$$

Discharge of one jet (q) = area of jet × velocity of jet

$$= \frac{\pi}{4}\times d^2\times V_1 = \frac{\pi}{4}\times(0.122)^2\times121.53 = 1.42m^3/s$$

$$\therefore \text{Number of jets required} = \frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{1.986}{1.42} = 1.4 \approx 2 \qquad \text{Ans.}$$

4.

A Francis turbine has an inlet diameter of 2.0m and an outlet diameter of 1.2m. The breath of the blades is constant at 0.2m. The runner rotates at a speed of

250rpm with a discharge of 8m³ per sec. The vanes are radial at the inlet and the discharge is radially outwards at the outlet. Calculate the angle of guide vane at the inlet and blade angle at the outlet.

Given data:

Inlet diameter, $D_1 = 2.0m$

Outlet diameter, $D_2 = 1.2m$

Breath diameter, $B_1 = B_2 = 0.2m$

Speed, $N = 250rpm$

Discharge, $Q = 8m^3/s$

☺ ***Solution:***

Flow velocity at inlet, $V_{f_1} = \frac{Q}{K_t \pi D_1 B_1} = \frac{8}{0.95 \times \pi \times 2 \times 0.2}$ ($\because$ Assume $K_t = 0.95$)

$$V_{f_1} = 6.7m/s$$

Also $V_{f_1} = K_f \sqrt{2gH}$

$$6.7 = 0.25 \times \sqrt{2 \times 9.81 \times H}$$

($\because K_f$ value varies from 0.15 to 0.3 Assume = 0.2:

$$H = 36.62m$$

Tangential velocity at inlet, $u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 2 \times 250}{60} = 26.18m/s$

We know that, Hydraulic efficiency

$$\eta_h = \frac{V_{w1} u_1}{gH}$$

Assume $\eta_h = 95\% = 0.95$

$$0.95 = \frac{V_{w_1} \times 26.18}{9.81 \times 36.61}$$

$$V_{w_1} = 13.03m/s$$

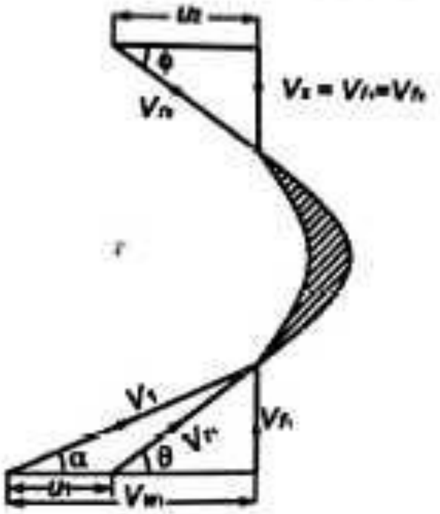


Figure 5.14

Angle of guide vane at inlet (α),

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{6.7}{13.03} = 0.514$$

$$\alpha = \tan^{-1}(0.514) = \mathbf{27.2°}$$ **Ans.**

We know that $$\frac{V_{f_1}}{V_{f_2}} = \frac{K_{t_2}\,\pi D_2\,B_2}{K_{t_1}\,\pi D_1\,B_1}$$ ($\because$ Assume $K_{t_1} = K_{t_2}$)

Since, $B_1 = B_2$ and $K_{t_1} = K_{t_2}$

$$\frac{V_{f_1}}{V_{f_2}} = \frac{D_2}{D_1}$$

$$\frac{6.7}{V_{f_2}} = \frac{1.2}{2.0}$$

$$V_{f_2} = 11.167 m/s$$

Tangential velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.2 \times 250}{60} = 15.71 m/s$$

Blade angle at the outlet (ϕ),

$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{11.167}{15.71} = 0.711$$

$$\phi = \tan^{-1}(0.711) = \mathbf{35.4^\circ}$$

5.

An inward flow turbine (Reaction turbine) with an overall efficiency of 80% is required to develop 150kW. Head is 8m. Peripheral velocity of wheel = 0.36 $\sqrt{2gH}$; Radial velocity of flow = 0.96 $\sqrt{2gH}$. Speed = 150rpm. Hydraulic losses = 22% of available energy. Assuming radial discharge determine (a) Guide blade angle at inlet. (b) Vane angle at outlet. (c) Diameter of wheel and (d) Width of wheel at inlet.

Given data:

Overall efficiency, η_o = 80% = 0.8

Power developed, P = 150kW

Head, H = 8m

Speed, N = 150rpm

Hydraulic losses = 22% of available energy

Discharge at outlet = Radial

$\therefore V_{w_2} = 0;\ V_{f_1} = V_{f_2}$

☺ ***Solution:***

Peripheral velocity of wheel, $u_1 = 0.36\ \sqrt{2gH}$

$$= 0.36\ \sqrt{2 \times 9.81 \times 8} = 4.51 m/s$$

Radial velocity of flow, $V_{f_1} = 0.96\ \sqrt{2gH} = 0.96\ \sqrt{2 \times 9.81 \times 8} = 12 m/s$

Hydraulic efficiency, $\eta_h = \dfrac{\text{Head at inlet} - \text{Hydraulic losses}}{\text{Head at inlet}}$

$$= \frac{H - 0.22H}{H} = \frac{0.78H}{H} = 0.78 = 78\%$$

We know that

$$\eta_h = \frac{V_{w_1} u_1}{gH}$$

$$0.78 = \frac{V_{w_1} \times 4.51}{9.81 \times 8}$$

$$V_{w_1} = 13.57 m/s$$

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{12}{13.57} = 0.884$$

The guide blade angle, $\alpha = \tan^{-1}(0.884) = \mathbf{41.48°}$ **Ans.**

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{12}{13.57 - 4.51} = 1.324$$

Vane angle at inlet, $\theta = \tan^{-1}(1.324) = \mathbf{53°}$ **Ans.**

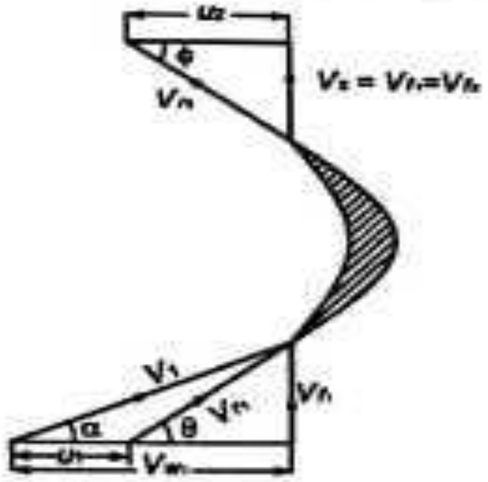


Figure 5.17

Velocity, $u_1 = \frac{\pi D_1 N}{60}$

$$4.51 = \frac{\pi \times D_1 \times 150}{60}$$

$$D_1 = 0.5742m = \mathbf{574.2mm}$$ A

Overall efficiency,

$$\eta_o = \frac{P}{wQH}$$

$$0.8 = \frac{150}{9.81 \times Q \times 8} \qquad (\because w = 9.81kN/m^3)$$

$$Q = 2.389m^3/s$$

We know that

$$Q = \pi D_1 B_1 V_{f_1}$$

$$2.389 = \pi \times 0.5742 \times B_1 \times 12$$

$$B_1 = 0.1103m = \mathbf{110.3mm}$$ A

A reaction turbine works at 450rpm under a head of 115m. The diameter of the inlet is 1.2m and the flow area is 0.4m². At the inlet, absolute and the relative velocities make angles of 20° and 60° respectively with the tangential velocity. Determine (i) The power developed (ii) Hydraulic efficiency. Assume the velocity of whirl at the outlet to be zero.

Given data:

Speed, $N = 450 rpm$

Head, $H = 115m$

Diameter at inlet, $D_1 = 1.2m$

Flow area, $A = 0.4m^2$

Angle made by absolute velocity, $\alpha = 20°$

Angle made by relative velocity, $\theta = 60°$

Solution:

Tangential velocity of the turbine,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.2 \times 450}{60} = 28.27 m/s$$

From inlet velocity triangle,

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}}$$

$$\tan 20° = \frac{V_{f_1}}{V_{w_1}}$$

$$V_{f_1} = V_{w_1} \times \tan 20° = 0.364 V_{w_1}$$

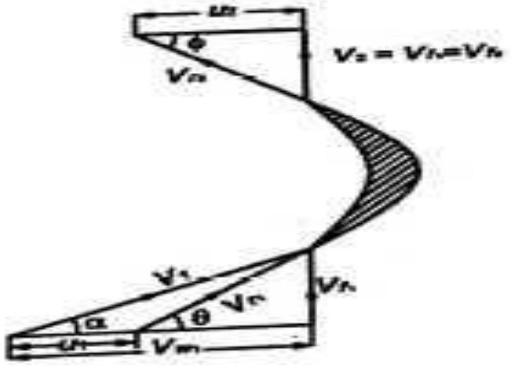


Figure 5.20

We know that, $\tan\theta = \dfrac{V_{f_1}}{V_{w_1} - u_1}$

$$\tan 60^\circ = \frac{0.364 V_{w_1}}{V_{w_1} - 28.27}$$

$$1.732 = \frac{0.364 V_{w_1}}{V_{w_1} - 28.27}$$

$$1.732 \left(V_{w_1} - 28.27\right) = 0.364 V_{w_1}$$

$$1.368\ V_{w_1} = 48.96$$

$$V_{w_1} = 35.789 m/s$$

$$\therefore\ V_{f_1} = 0.364 \times 35.789 = 13 m/s$$

Volume flow rate, $Q = A \times V_{f_1} - 0.4 \times 13 = 5.2 m^3/s$

Power developed, $P = w\, Q\, H$

$= 9.81 \times 5.2 \times 115 =$ **5866.38kW** **Ans.**

Hydraulic efficiency, $\eta_h = \frac{V_{w_1}\ u_1}{gH} = \frac{35.789 \times 28.27}{9.81 \times 115} = 0.897 =$ **89.7%** **Ans.**

7.

An outward flow reaction turbine has internal and external diameters of the runner as 0.5m and 1.0m respectively. The turbine is running at 250rpm and rate of flow of water through the turbine is 8m³/sec. The width of the runner is constant at inlet and outlet and is equal to 30cm. The head on the turbine is 10m and discharge at outlet is radial, determine (i) Vane angle at inlet and outlet. (ii) Velocity of flow at inlet and outlet. ***[Nov'03]***

Given data:

Inlet diameter, $D_1 = 0.5m$

Outlet diameter, $D_2 = 1.0m$

Breath diameter, $B_1 = B_2 = 30cm = 0.3\ m$

Head, $H = 10m$

Speed, $N = 250rpm$

Discharge, $Q = 8m^3/s$

☺ ***Solution:***

Flow velocity at inlet, $V_{f_1} = \dfrac{Q}{K_t \pi D_1 B_1}$

$$= \frac{8}{0.95 \times \pi \times 0.5 \times 0.3} \qquad (\because \text{Assume } K_t = 0.95)$$

$$V_{f_1} = \mathbf{17.87m/s}$$

Ans.

Tangential velocity at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.5 \times 250}{60} = 6.54 m/s$$

We know that,

Hydraulic efficiency,

$$\eta_h = \frac{V_{w1} u_1}{gH}$$

Assume $\eta_h = 95\% = 0.95$

$$0.95 = \frac{V_{w_1} \times 6.54}{9.81 \times 10}$$

$$V_{w_1} = 14.25 m/s$$

Angle of guide vane at inlet (α),

$$\tan\alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{17.87}{14.25} = 1.254$$

$$\alpha = \tan^{-1}(1.254) = \mathbf{51°25'}$$ **Ans.**

We know that,

$$\frac{V_{f_1}}{V_{f_2}} = \frac{K_{t_2}\, \pi D_2 B_2}{K_{t_1}\, \pi D_1 B_1}$$ ($\because$ Assume $K_{t_1} = K_{t_2}$)

Since, $B_1 = B_2$ and $K_{t_1} = K_{t_2}$

$$\frac{V_{f_1}}{V_{f_2}} = \frac{D_2}{D_1}$$

$$\frac{17.87}{V_{f_2}} = \frac{0.5}{1.0}$$

Velocity of flow at outlet, $V_{f_2} = \mathbf{35.74 m/s}$ **Ans.**

Tangential velocity at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.0 \times 250}{60} = 13.09 m/s$$

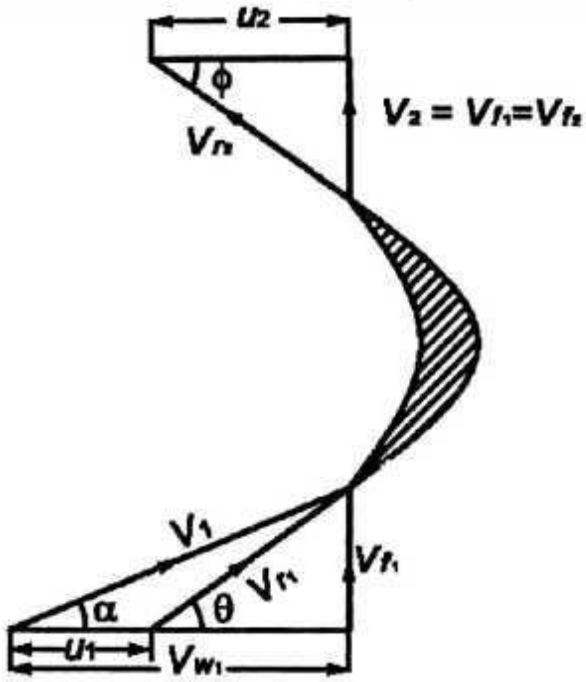


Figure 5.31

Blade angle at the outlet (φ),

$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{35.74}{13.09} = 2.73$$

$$\phi = \tan^{-1}(2.73) = \mathbf{69°53'}$$